Auto–information in non–Markovian diffusion systems.

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Mauro Faccin

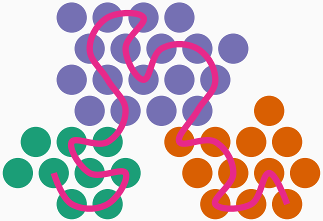
icteam, UCLouvain, Belgium

Intro

Projected Markov Chains

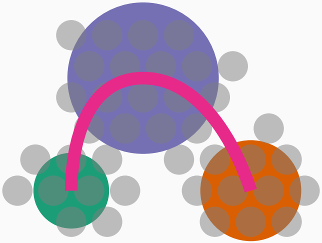
A markov chain

$$\ldots, x_{\text{past}}, x_{\text{now}}, x_{\text{future}}, \ldots$$

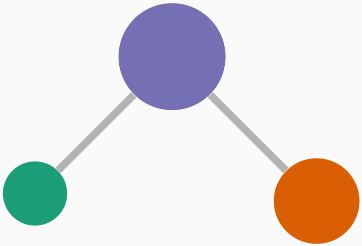


Its projection

$$\ldots, y_{\text{past}}, y_{\text{now}}, y_{\text{future}}, \ldots$$



Where did the complexity go?



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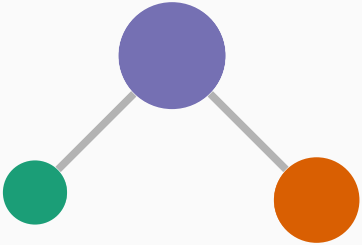
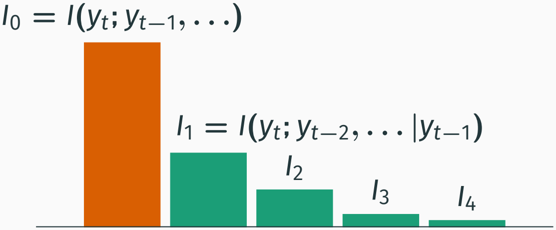


> Part of the complexity is now hidden in the [projected] dynamics.

> Emergence of **effective memories**.

The Entrogram

Information flowing from the **PAST** toward the **FUTURE**.

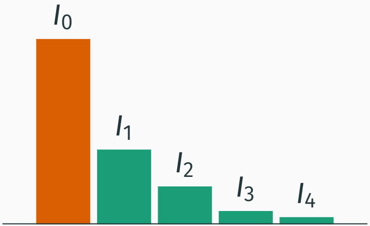


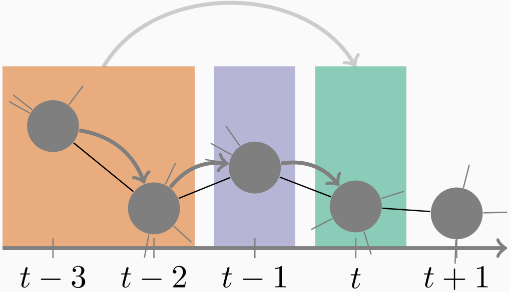
where $I(X;Y) = H(X) - H(X|Y)$ is the Mutual Information

Faccin, Schaub, Delvenne Journal of Complex Networks, 6(5), 2018, p661–678

Entrogram: Information flowing from the PAST to the FUTURE

$$I(y_t;\, y_{t-2}, y_{t-3}, \dots | y_{t-1})$$



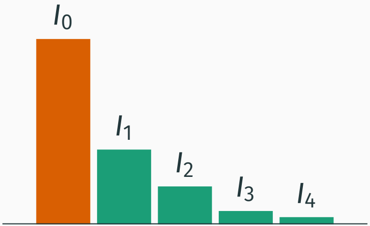


Faccin, Schaub, Delvenne Journal of Complex Networks, 6(5), 2018, p661–678

Crutchfield and Young (1989) PRL, 63, 105.

Crutchfield and Feldman (2003) Chaos, 13, 25–54.

Entrogram: a compact description of the system complexity



> [Total] Predictability, how the dynamics are aligned to the partition.

> Emergent effective memory

> \+ Overall complexity (excess entropy) of the dynamical process

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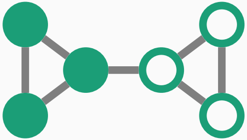
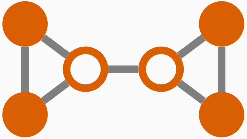
An Example: the Bow–tie

Assortative partition



Equitable partition



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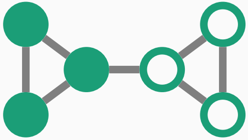
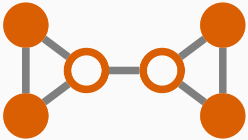
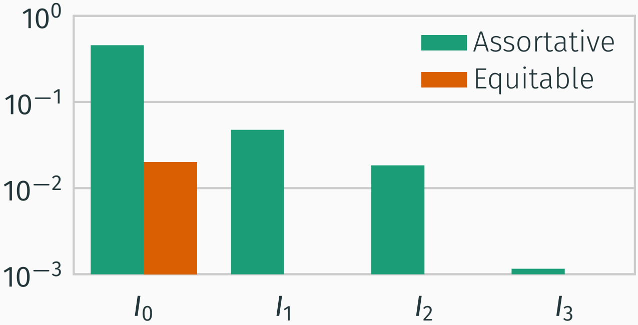


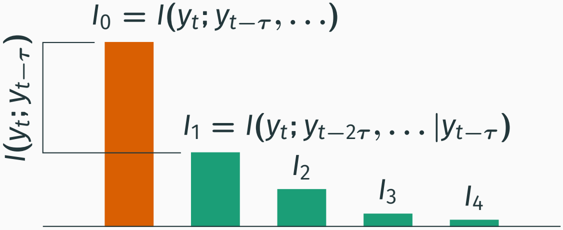
Equitable partition





Auto–information

Enhance your Entrogram



where τ represents a time–scale parameter.

M.F. et al, Journal of Complex Networks, cnx055

Modularity:

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j)$$

χ_c characteristic variable of partition c

$$Q \propto \sum_{c} \mathbf{Cov}\left(\chi_c(t), \chi_c(t+1)\right)$$

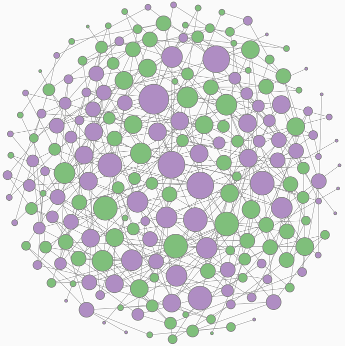
Modularity can be interpreted as auto–covariance of the dynamics (linear dependence of consecutive time-steps).

Shen et al. (2010) PRE, 82, 016114

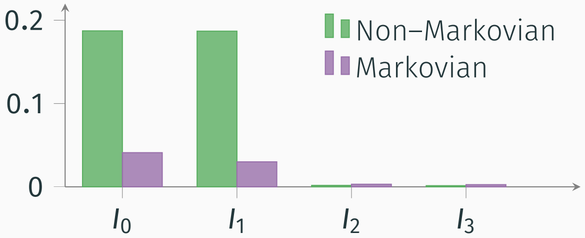
Blocks and NON–Markovian dynamics

Example: Backtracking

Erdős–Rényi with nodes of two different classes.



Walkers have higher probability to *go back* to a visited class (second order memory).



The complexity of the system resides on the dynamics.

Example: Backtracking

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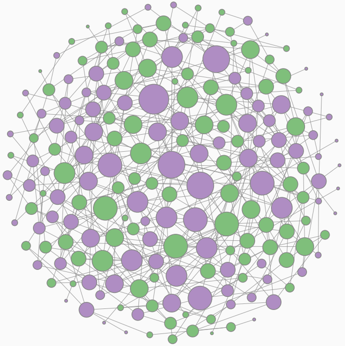

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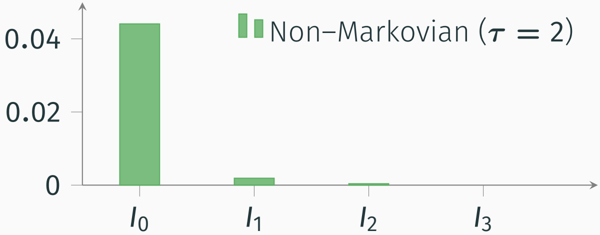

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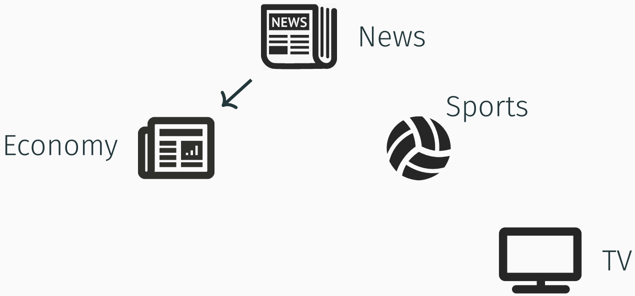
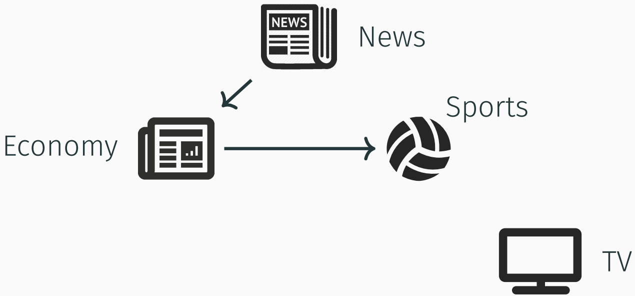
Web–portal

Web–portal



Web–portal



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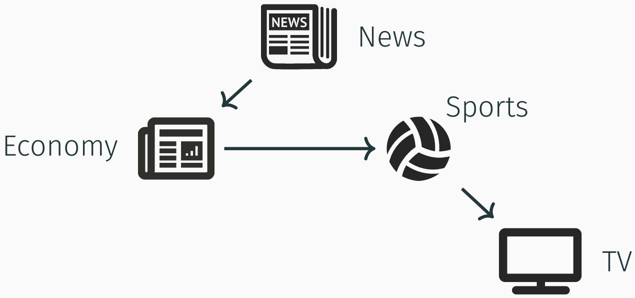
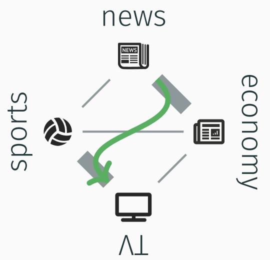
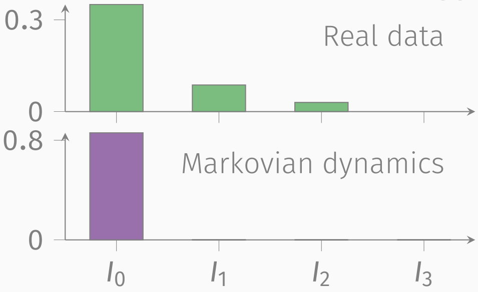


Web–portal users are non–Markovian

Web portal click behaviour

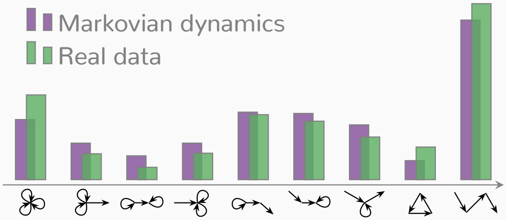


Single recorded path:



Comparison of the entrogram for the real dataset and the Markovian approximation. The former is at least Markovian of third order.

Web–portal: dynamical patterns

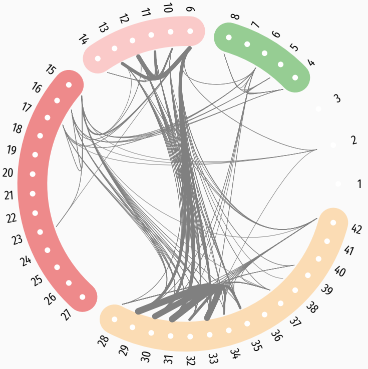


Some dynamical patterns are underestimated by the Markovian approximation.

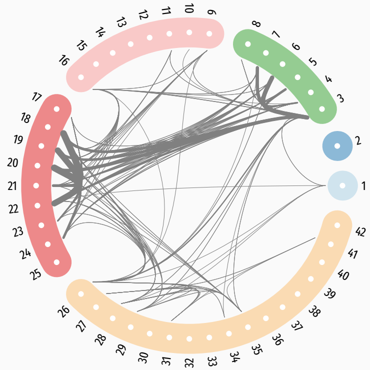
3–steps dynamical patterns of the original dataset (green) and the Markovian approximation (purple).

Web–portal: page partition

Short time–scale

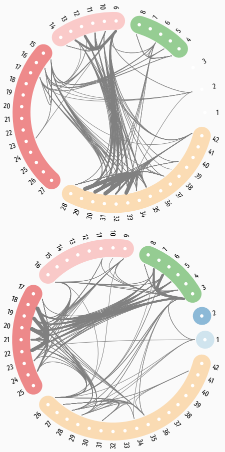


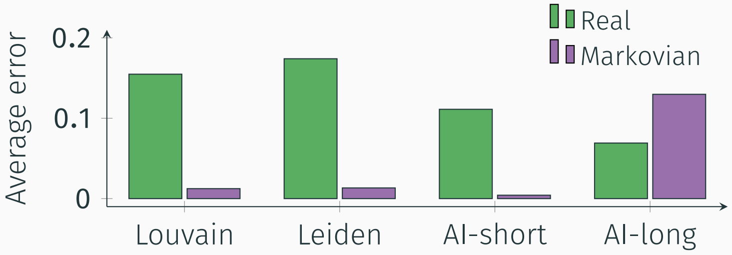
Longer time–scale ($\tau = 3$)



Partitions with different time–scales are slightly different.

Web–portal: dynamics simulations

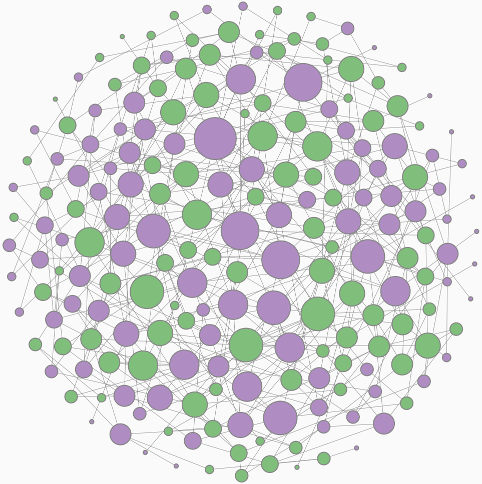




Average error after few steps (6) when using the partitions as a Markovian model.
Comparison with the original dynamics (green) and the Markovian approximation (purple).

Conclusions

Questions?



Joint work with:

JC Delvenne

icteam

UCLouvain

M Schaub

UNIVERSITY OF OXFORD

IDSS

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https://maurofaccin.github.io
mauro.faccin@uclouvain.be
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Code at:

```
https://github.com/maurofaccin/entropart
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The full story

We need a regularization term:

$$\mathcal{I}_\beta(\tau) = I(y_t, y_{t-\tau}) - \beta H(y_t)$$