Variable aggregation for dynamical systems on networks.

Mauro Faccin

Benet 2019 (Hasselt University)

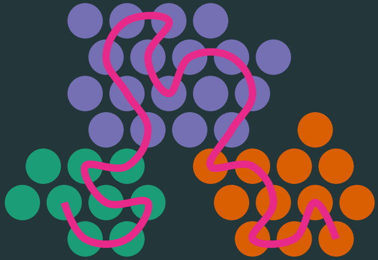
Projected Markov Chain

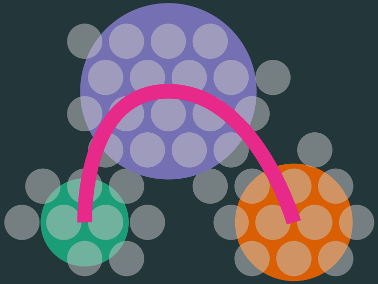
Markov Chain

$$\ldots, x_{\text{past}}, x_{\text{now}}, x_{\text{future}}, \ldots$$

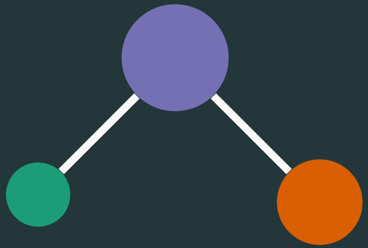
Projection

$$\ldots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \ldots$$





Where did the complexity go?



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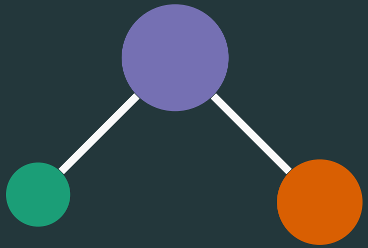
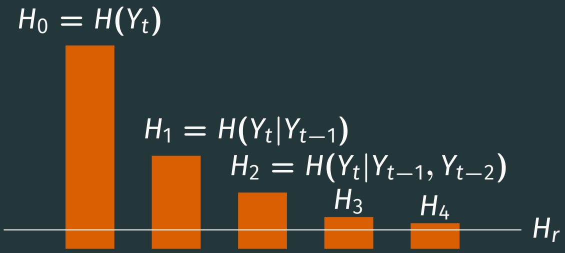


Part of the complexity is now hidden in the [projected] dynamics.

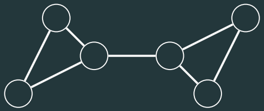
Emergence of **effective memories**.

Effective Memory

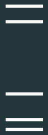
How can we compute the emerfing memory effects?



Eigen Partitions



Eigenvalues



Eigen Partitions

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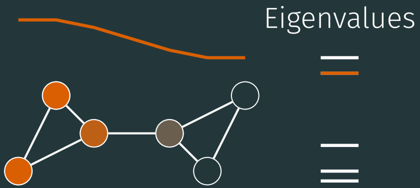
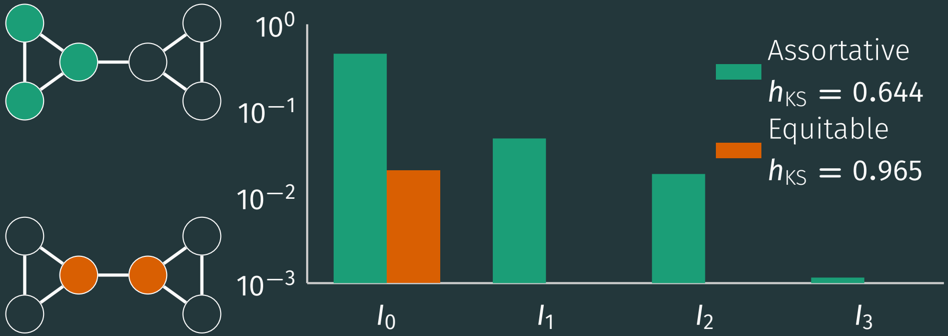


Eigen Partitions

Bow tie



10^0
10^{-1}
10^{-2}
10^{-3}
l_0
l_1
l_2
l_3
Assortative
$h_{KS} = 0.644$
Equitable
$h_{KS} = 0.965$

Partitioning with dynamics

Random walk covariance

A walker is visiting nodes on the network. Let's measure how it get trapped in a partition.

Let's define:

χ_c characteristic function of partition c

Covariance of partitions along the dynamics:

$$\mathbf{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$E(\chi_c(t)\chi_c(t-1)) = \frac{1}{2m}\sum_{ij \in c} A_{ij}$$

$$E(\chi_c(t)) = \frac{1}{2m}\sum_{i \in c} k_i$$

Modularity:

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j)$$

Random walker covariance

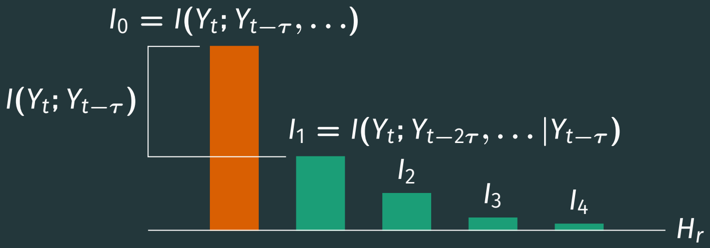
χ_c characteristic function of partition c

$$Q \propto \sum_c \mathbf{Cov}\left(\chi_c(t), \chi_c(t+1)\right)$$

Linear correlation between consecutive time-steps.

Shen et al. (2010) PRE, 82, 016114

Back to the Entrogram



where τ represents a time-scale parameter.

M.F. et al, Journal of Complex Networks, cnx055

What about generative models?

$$I(Y_t; Y_{t-1}) = H(Y_y) + H(Y_{t-1}) - H(Y_y, Y_{t-1})$$

$$H(Y_t) = -\sum_c \frac{e_c}{2m} \log \frac{e_c}{2m} \qquad e_c = \sum_{i \in c, j} A_{ij}$$

$$H(Y_t, Y_{y-1}) = -\sum_{cd} \frac{e_{cd}}{2m} \log \frac{e_{cd}}{2m} \qquad e_{cd} = \sum_{i \in c, j \in d} A_{ij}$$

In binary undirected networks

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DC-SBM

$$\mathcal{S}_C \propto -\frac{1}{2}\sum_{cd} e_{cd} \log \frac{e_{cd}}{e_c e_d}$$

In binary undirected networks

Non linear communities

Objective function: $I(Y_t, Y_{t-\tau})$

Modularity

$Q \propto \sum_c \mathbf{Cov}\left(\chi_c(t), \chi_c(t+1)\right)$

Spectral Properties

DC-SBM

$I(Y_t; Y_{t-\tau}) \propto -\sum_{rs} e_{rs} \log \frac{e_{rs}}{e_r e_s}$

In some cases

Shen et al. (2010) PRE, 82, 016114.

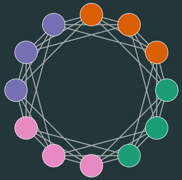
Karrer and Newman (2011), PRE 83, 016107.

Rosvall and Bergstrom (2008) PNAS 105, 1118.

Where the dynamical part enter?

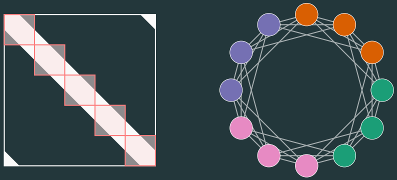
How many partitions?

Adj:



How many partitions?

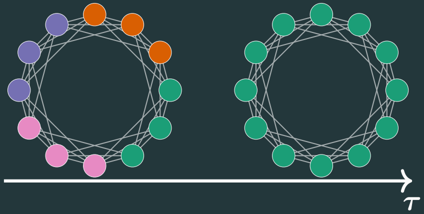
Adj:



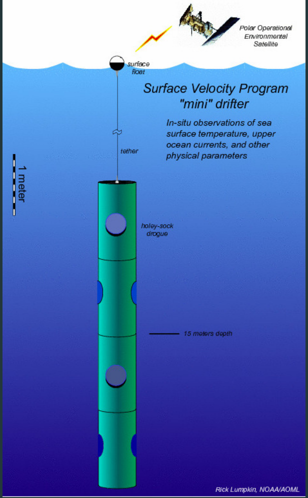
Example 1: One cycle

How many partitions?

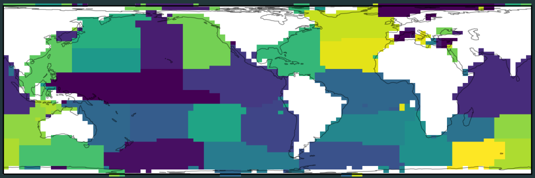
Adj:



Example 3. Ocean buoys



$\tau = 7$ days



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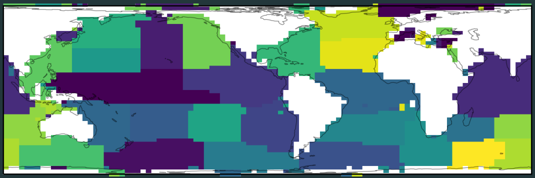

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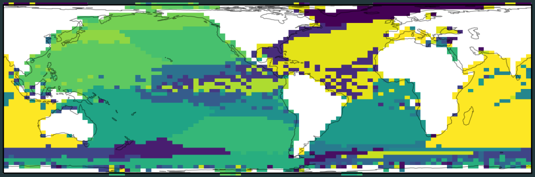

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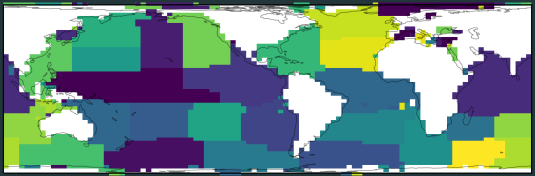

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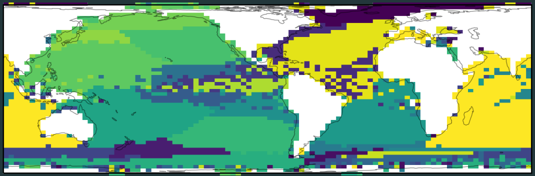

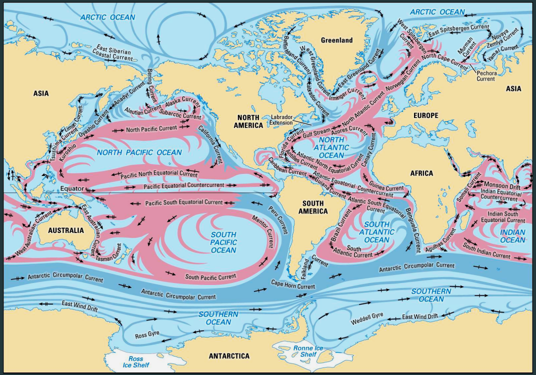

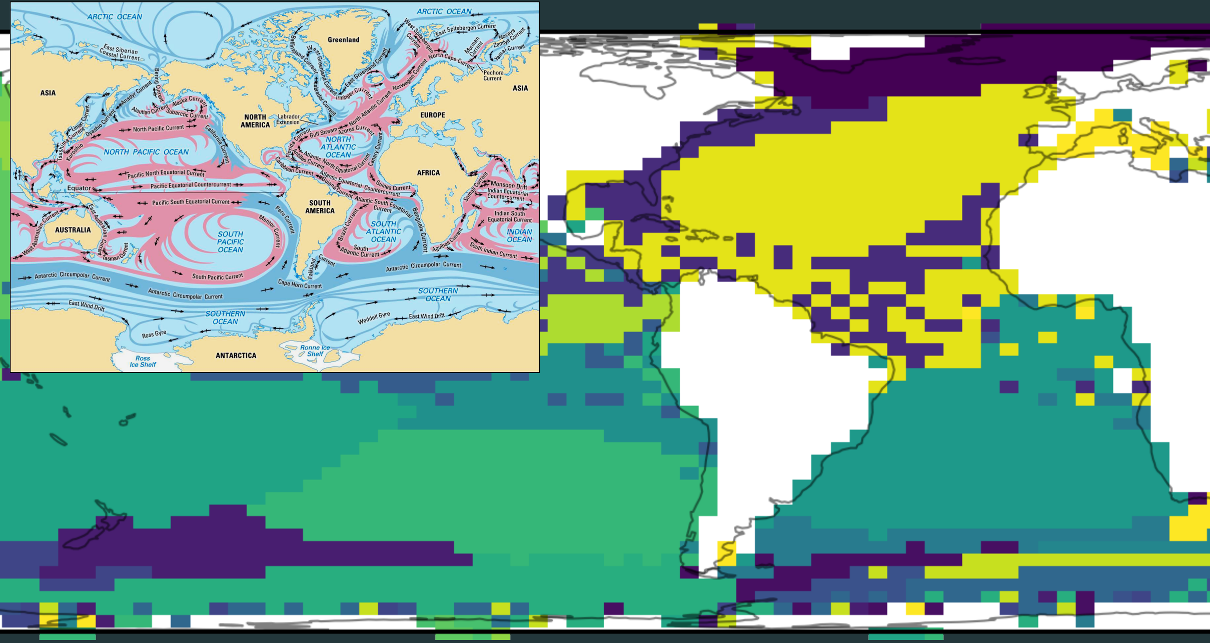
ARCTIC OCEAN
ARCTIC OCEAN
Greenland
East Siberian Coastal Current
West Spitsbergen Current
East Spitsbergen Current
Novaya Zemlya Current
Murman Current
Yamal Current
North Cape Current
Pechora Current
Baffin Island Current
West Greenland Current
East Greenland Current
Labrador Current
Irminger Current
Norwegian Current
North Atlantic Current
ASIA
ASIA
Bering Current
Anadyr Current
Alaska Current
Aleutian Current
Subarctic Current
Oyashio Current
Liman Current
Tsushima Current
Kuroshio
North Pacific Current
California Current
NORTH AMERICA
Labrador Extension
EUROPE
Florida Current
Gulf Stream
Azores Current
NORTH ATLANTIC OCEAN
Antilles Current
Atlantic North Equatorial Current
Canary Current
NORTH PACIFIC OCEAN
Caribbean Current
AFRICA
Pacific North Equatorial Current
Pacific Equatorial Countercurrent
Guiana Current
Atlantic Equatorial Countercurrent
Guinea Current
Somali Current
Monsoon Drift
Indian Equatorial Countercurrent
Equator
Pacific South Equatorial Current
Atlantic South Equatorial Current
Indian South Equatorial Current
SOUTH AMERICA
Peru Current
Mentor Current
Brazil Current
Benguela Current
AUSTRALIA
East Australian Current
West Australian Current
Tasman Current
SOUTH PACIFIC OCEAN
SOUTH ATLANTIC OCEAN
INDIAN OCEAN
Agulhas Current
South Atlantic Current
South Indian Current
Falkland Current
Antarctic Circumpolar Current
Antarctic Circumpolar Current
Antarctic Circumpolar Current
South Pacific Current
Cape Horn Current
SOUTHERN OCEAN
SOUTHERN OCEAN
East Wind Drift
East Wind Drift
Weddell Gyre
Ross Gyre
Ronne Ice Shelf
Ross Ice Shelf
ANTARCTICA

Questions?

FIG. 173.—FRANKLIN'S CHART OF THE GULF STREAM.

Joint work with:

JC Delvenne

M Schaub

https://maurofaccin.github.io
mauro.faccin@uclouvain.be

Code at:
https://github.com/maurofaccin/entropart