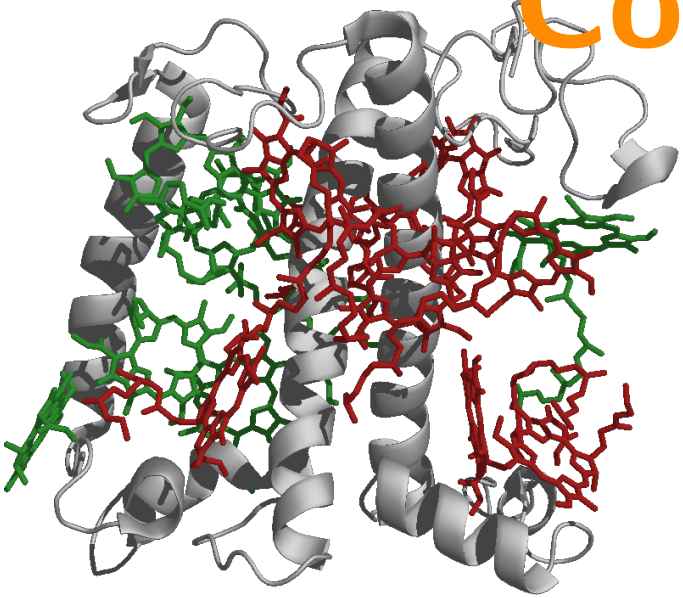


Community Detection in Quantum Systems

Mauro Faccin

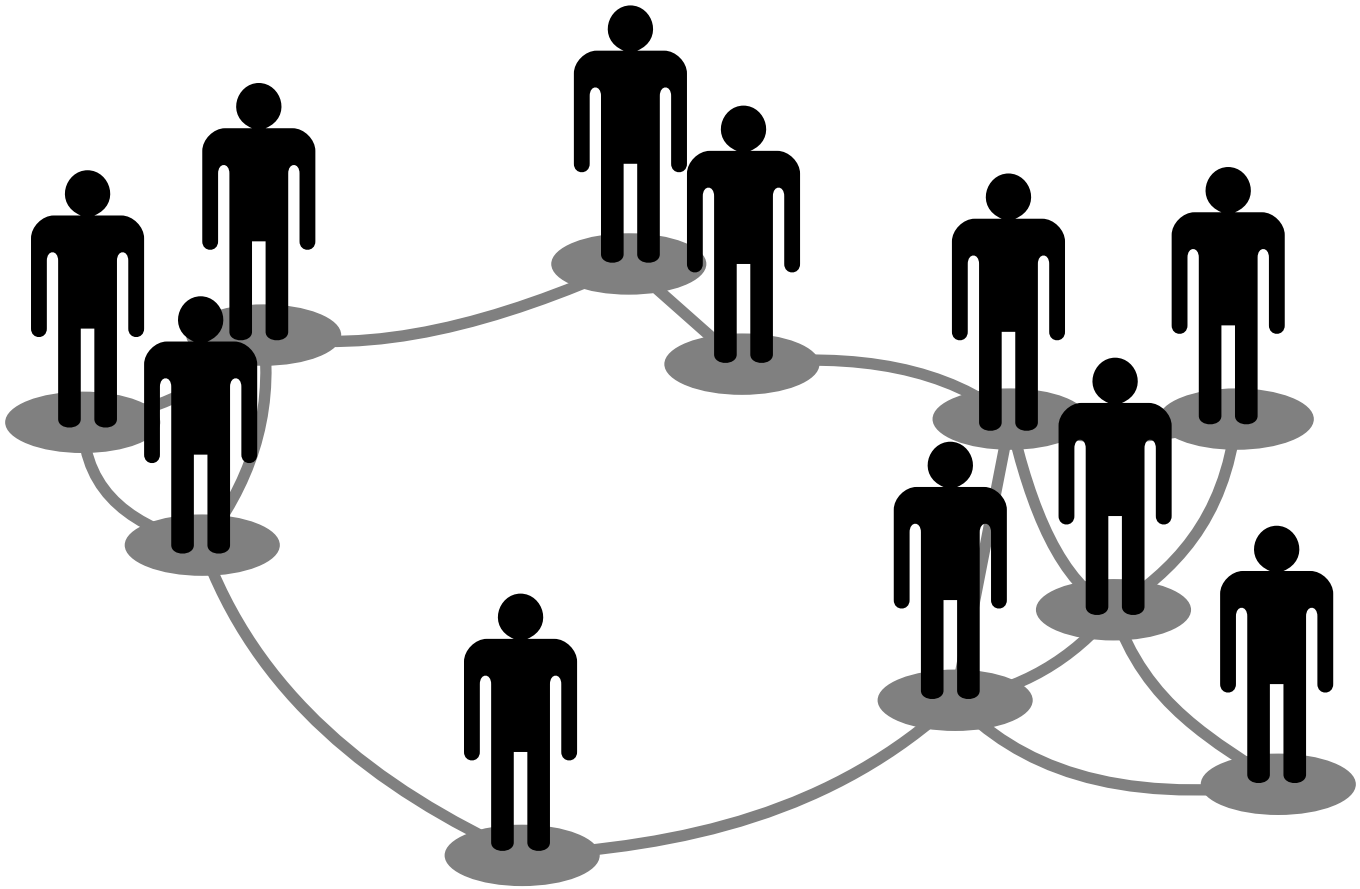
ISI Foundation

BIFI

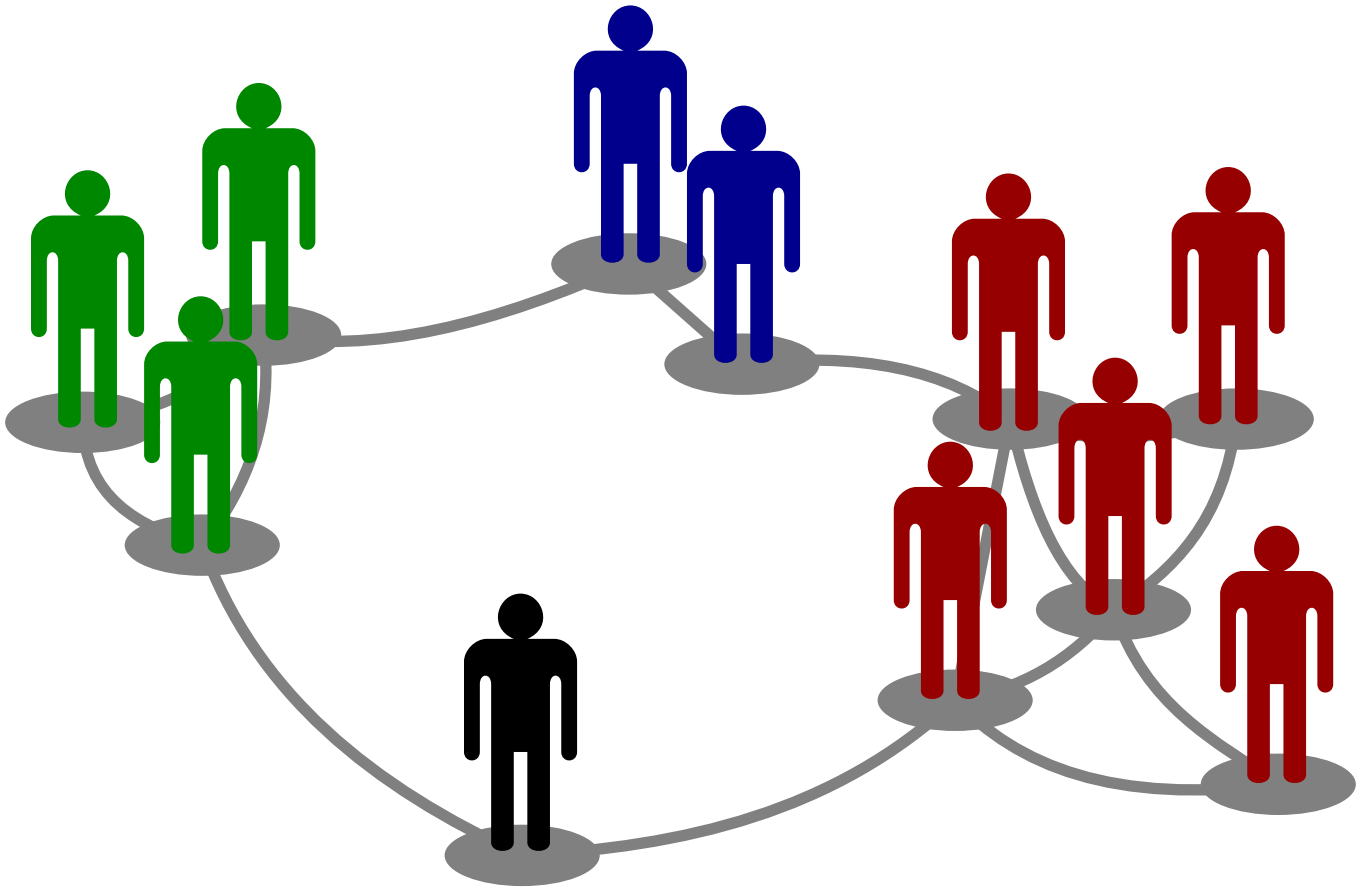
BIFI2014

Zaragoza, January 22-24, 2014

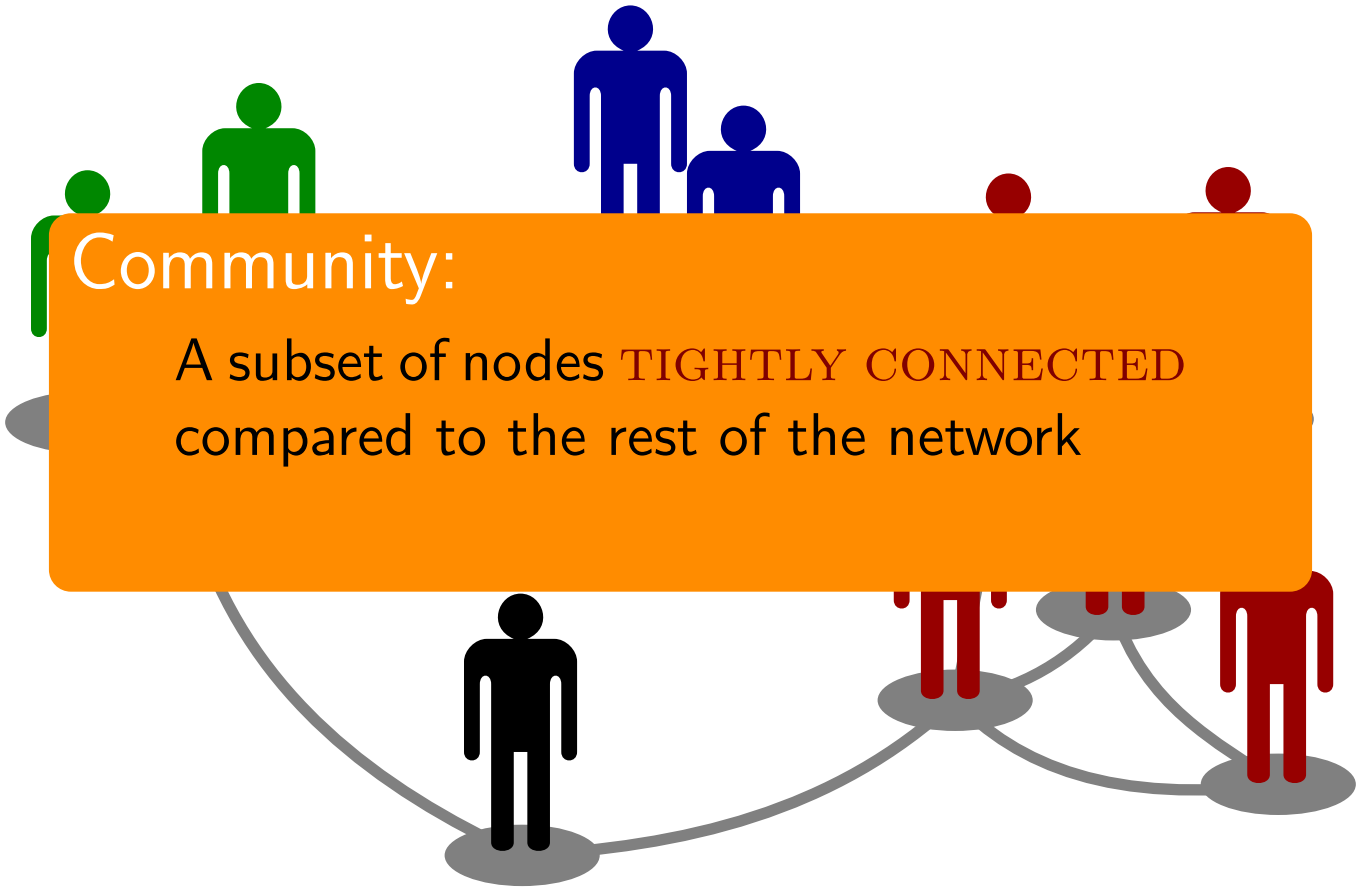
Communities: an overview



Communities: an overview



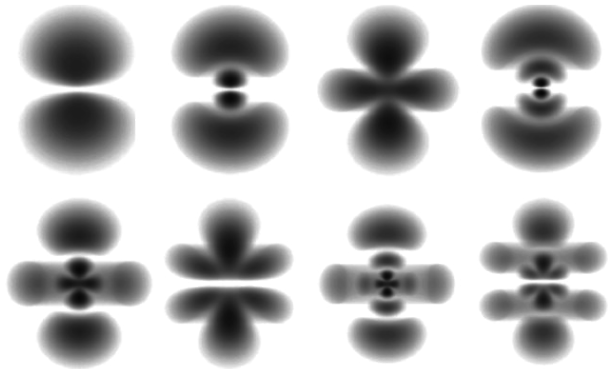
Communities: an overview



Quantum Systems: an overview

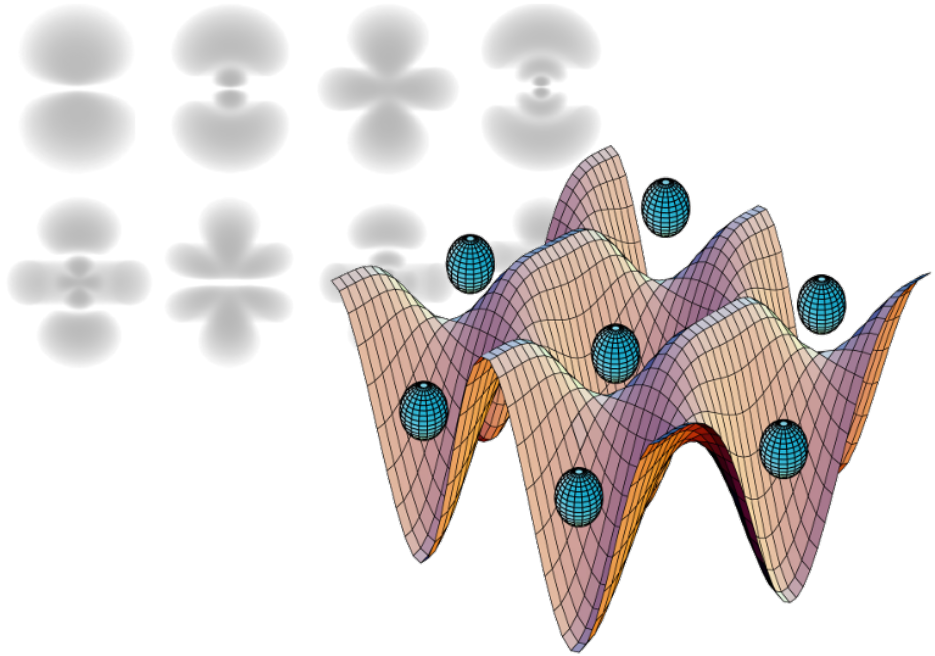
Quantum Systems: an overview

- Simple Systems (Atomic Orbitals...)



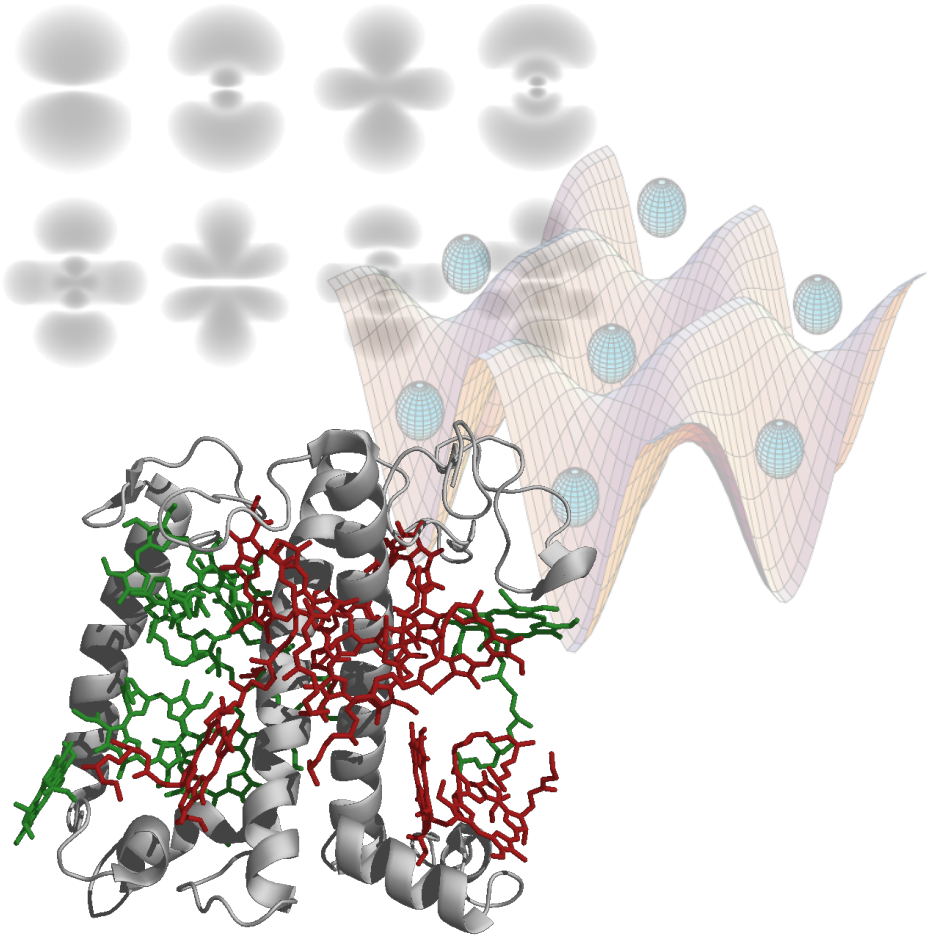
Quantum Systems: an overview

- Simple Systems (Atomic Orbitals...)
- Symmetric Systems (Lattices...)



Quantum Systems: an overview

- Simple Systems (Atomic Orbitals...)
- Symmetric Systems (Lattices...)
- Complex Systems (Proteins...)

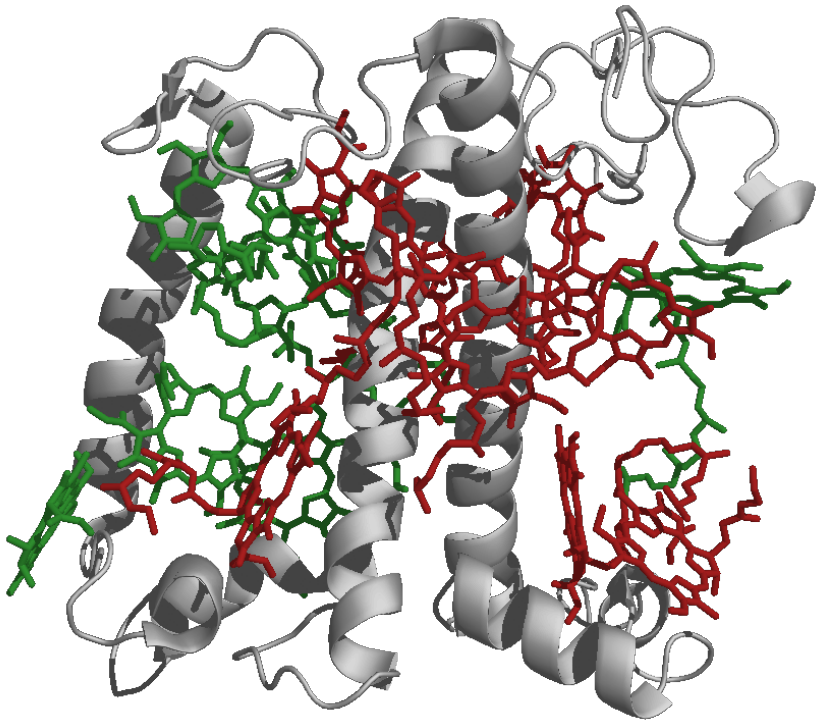


Quantum Mechanics & Complex Networks

- Quantumness on quantum walks (relation with degree distribution)
- Chiral walks (new effects in quantum walks)
- Quantum internet (Quantum Google, . . .)

Communities in Quantum Systems: Why?

- Numerics
- Quantum Computing
- ...



Community Detection, the Usual Way

- Network characterization (eigenvalues, edges ranking. . .)
- Statistical significance
- Random walker: stay trapped in a community
- . . .

Available methods:

Girvan and Newman, Modularity optimization, Spin models, Clique percolation, Spectral methods, OSLOM, InfoMap, COPrA, Stability . . .

What's a Quantum Community?

A quantum community should show "quantum behavior" such as:

- Interference
- Coherence
- Entanglement
- ...

Our approach:

- Quantum walks on a graph (one particle subspace)
- Define a closeness measure between nodes (should include *quantum behavior*)
- Based on node closeness find communities (hierarchical clustering, modularity maximization)

Quantum Community: Transport Approach

Transfer matrix:

$$T_{ij}(t) = |\langle i|\mathrm{e}^{-\mathrm{i}Ht}|j\rangle|^2$$

Minimize the probability that a quantum walker leave a community

Closeness:

$$c_t^T(A,B) = \frac{1}{|A||B|}\sum_{i\in A, j\in B} T_{ij}(t) + T_{ji}(t)$$

Two communities A and B are close if the transport flow between them is high.

Quantum Community: Fidelity Approach

Fidelity:

$$F(\rho, \sigma) = |\langle\psi|\phi\rangle|$$

for pure states:

$$\rho = |\psi\rangle\langle\psi|$$
$$\sigma = |\phi\rangle\langle\phi|$$

Maximize the coherece between nodes within a community.

Closeness:

$$c_t^F(A, B) = \frac{\mathcal{F}_A(t) + \mathcal{F}_B(t) - \mathcal{F}_{A\cup B}(t)}{|A||B|}$$

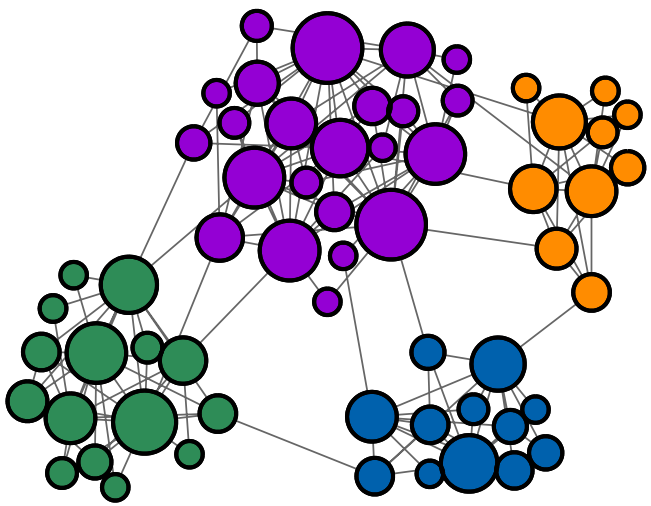
where

$$\mathcal{F}_X(t) = F^2(\rho_A(t), \rho_A(0))$$

Two communities A and B are close if the coherence between them persists.

An Example

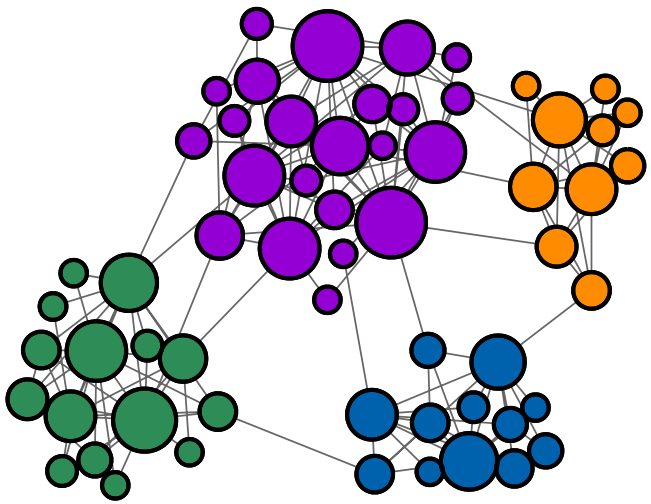
Benchmark network:



Lancichinetti et al., PRE, 78:046110, 2008.

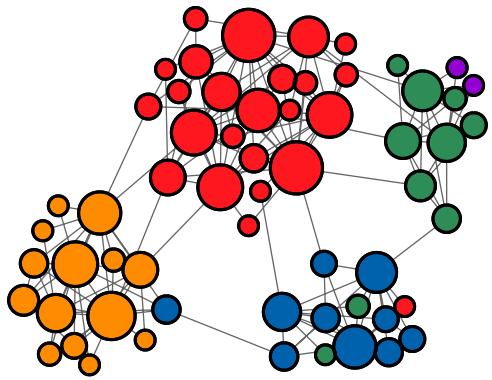
An Example

Benchmark network:

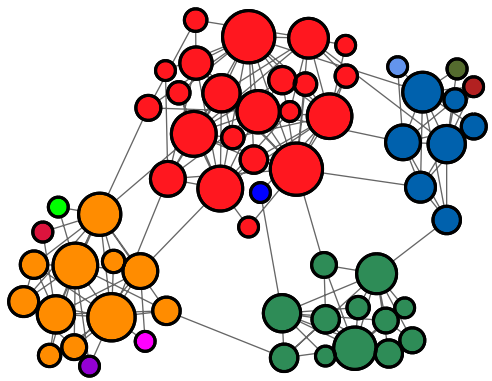


Lancichinetti et al., PRE, 78:046110, 2008.

Transport



Fidelity



An Example

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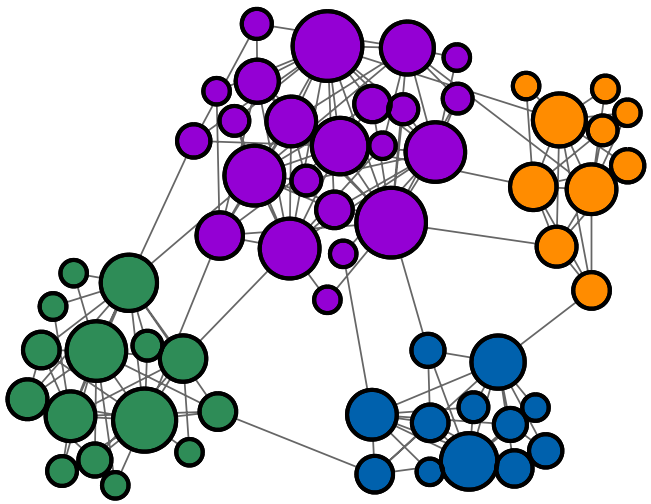

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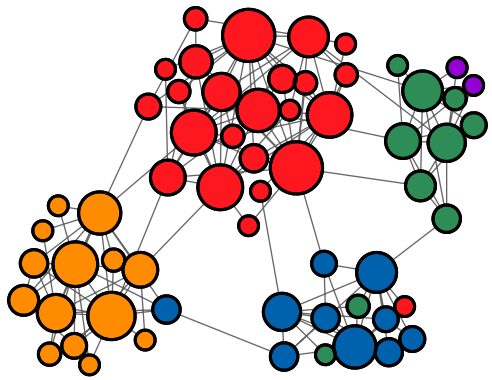
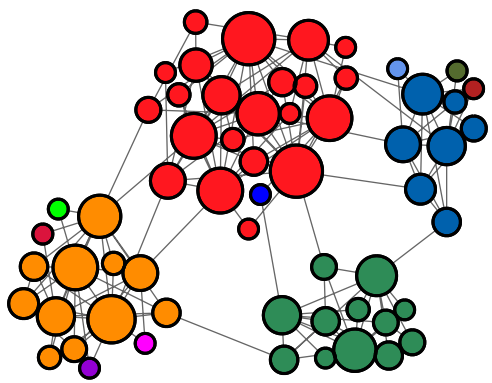
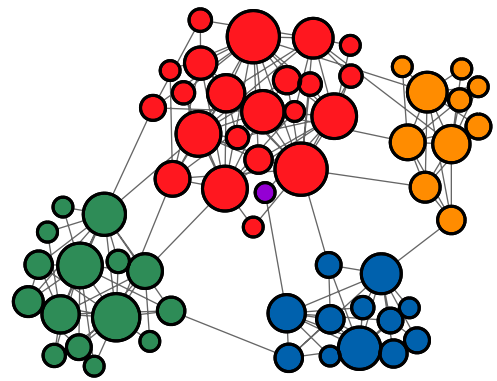


Fidelity

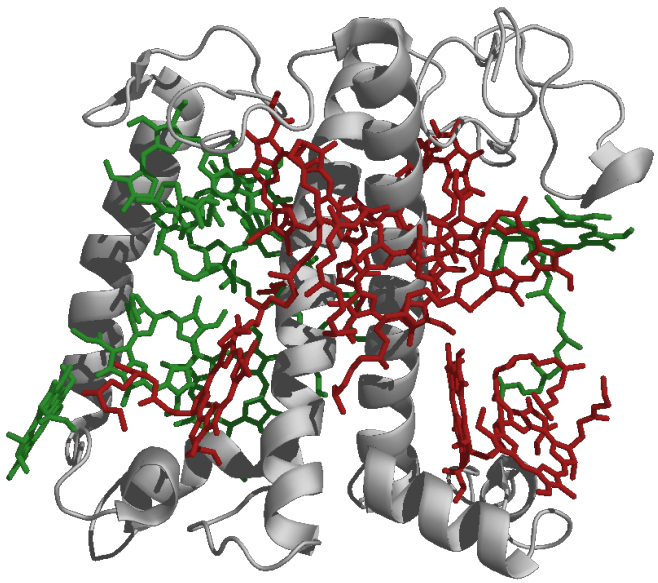


OSLOM

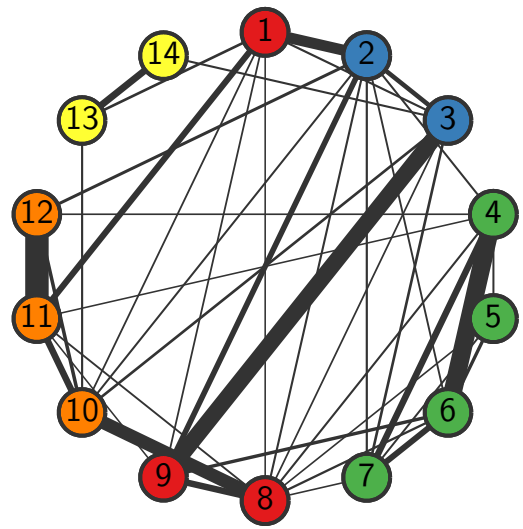


LHCII

Light harvesting systems LHCII:

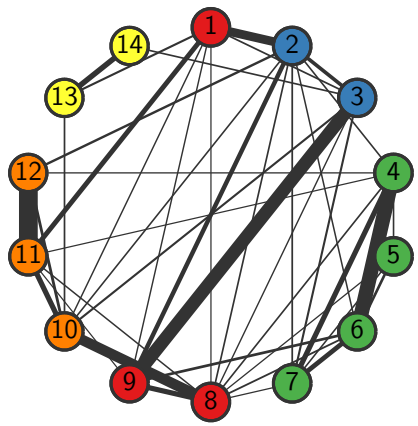


14 nodes network with Hamiltonian H.



LHCII

Light harvesting systems LHCII:



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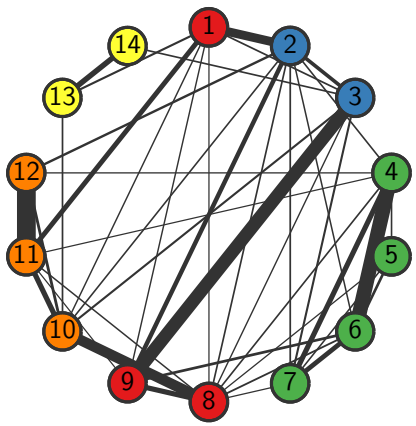
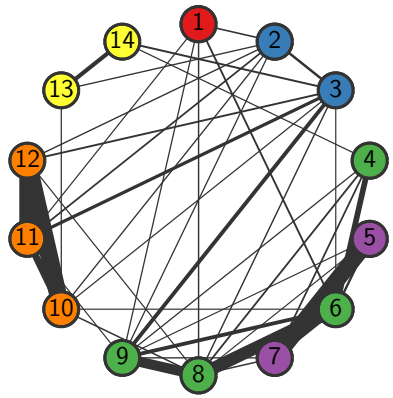
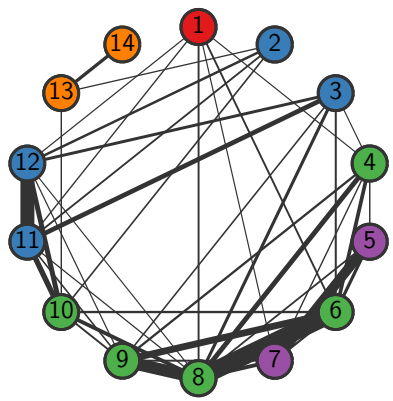


Transport



Fidelity



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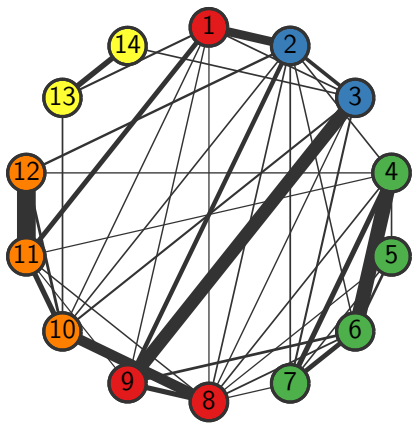
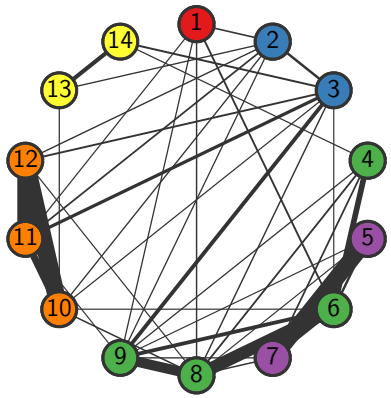
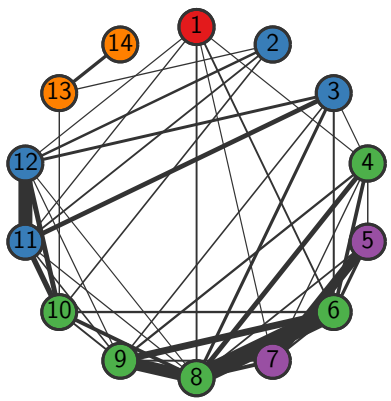


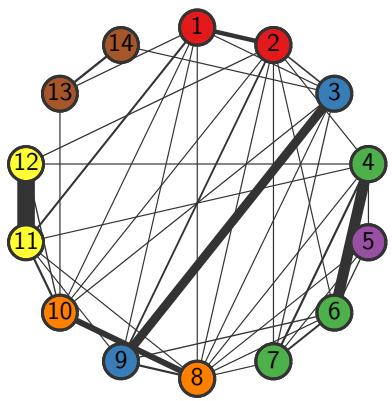
Transport



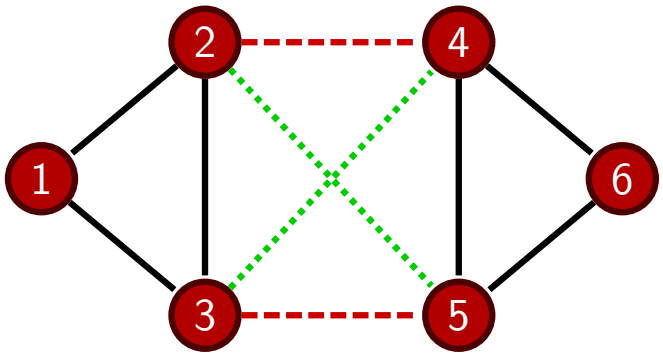
Fidelity



Transport (short time)

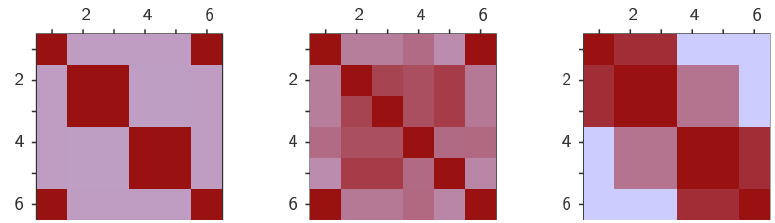


Understanding

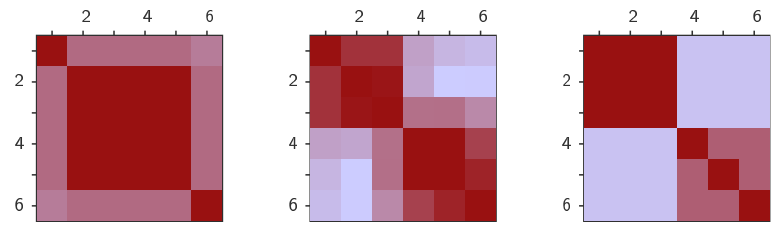


Toy graph to reveal non-intuitive *quantum effects*

Phases' effect on transport:



Phases' effect on fidelity:



All ones

Random

Cancelling

Comments

- A community detection algorithm based on quantum mechanics
- Community detection FOR quantum systems
- New "Quantum Network" field

Ads: arXiv:1310.6638

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www.TheQuantumNetwork.org