# Beyond Communities: Dynamical Markov Modules

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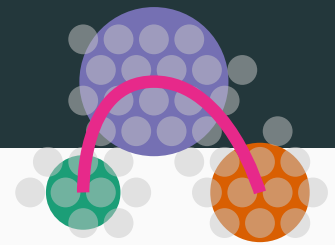
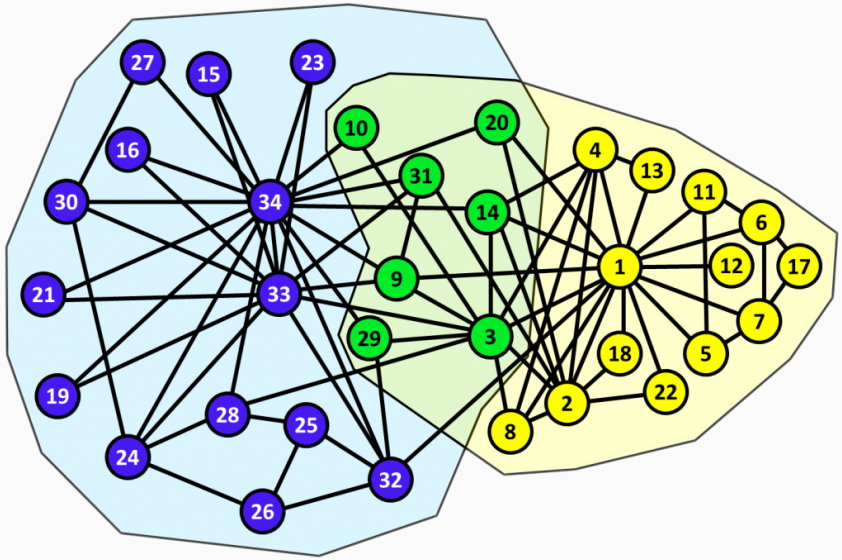
# Partitioning Problem



## Why should we care

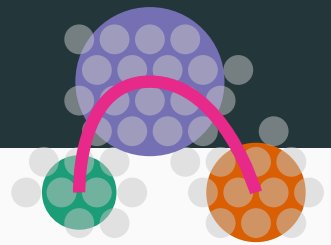
- A simple description of the structure
- A model for the system dynamics
- Insight




What about other link patterns?

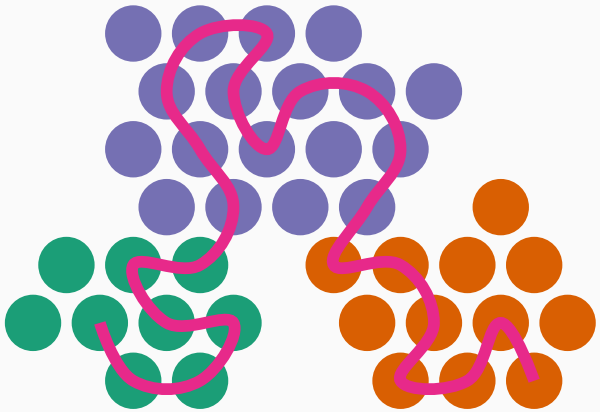
# Modules defined by dynamics

# Projected Markov Chain



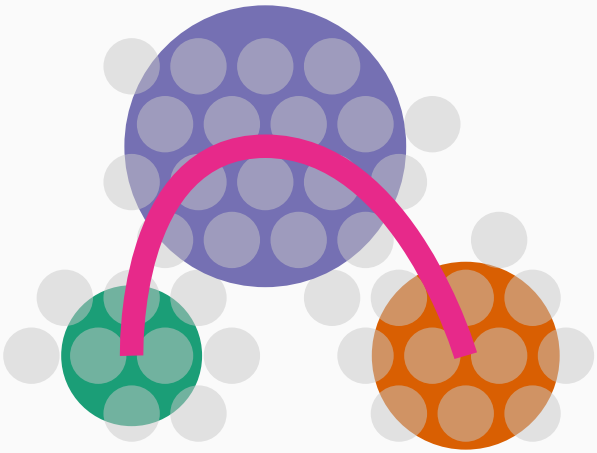
## Markov Chain

$$\ldots, x_{\text{past}}, x_{\text{now}}, x_{\text{future}}, \ldots$$

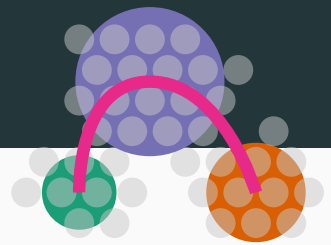


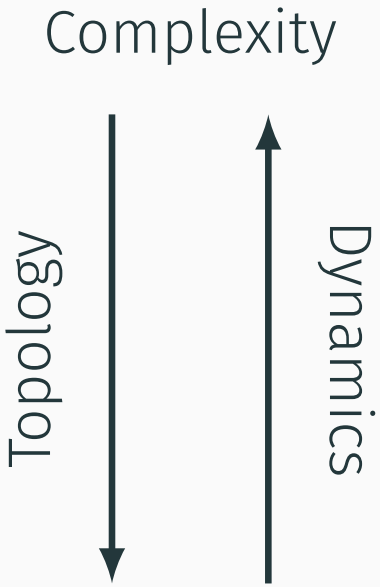
## Projection

$$\ldots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \ldots$$

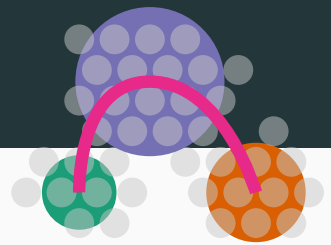


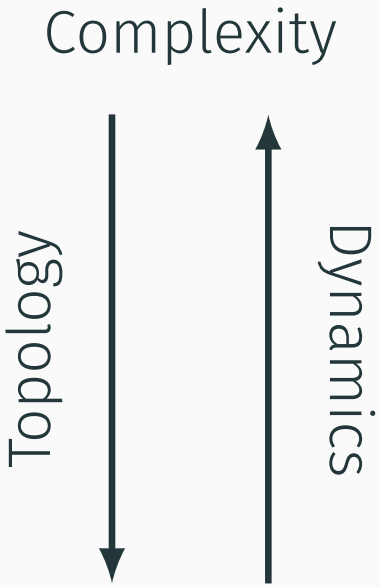
# Complexity





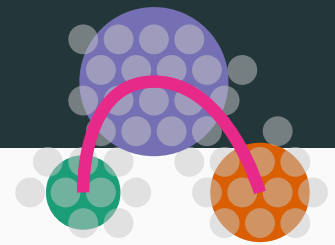
# Complexity






Memories

# Assess the Partition Quality 1



Projected Markov Chain:

$$\ldots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \ldots$$

**Maximize predictability:**

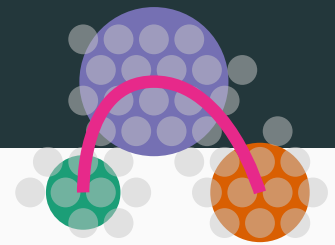
$$I(Y_{\text{future}}; Y_{\text{past}})$$

amount of information flowing from *past* to *future*.

$$I(Y_{\text{future}}; Y_{\text{past}}) \leq I(x_{\text{future}}; x_{\text{past}})$$

- Increases predictability of future with knowledge of past;
- Favors heterogeneous module-linking;
- Favors homogeneous module size;
- $I(\cdot;\cdot)$ is Mutual Information

# Assess the Partition Quality 2



Projected Markov Chain:

$$\ldots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \ldots$$

**Minimize memories:**

$$I(Y_{\text{future}}; Y_{\text{past}} | Y_{\text{now}})$$

higher order memory embedded into the process.

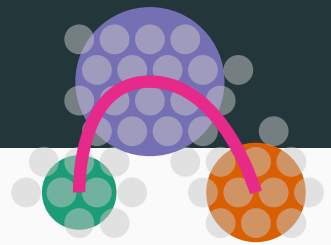
$$I(Y_{\text{future}}; Y_{\text{past}} | Y_{\text{now}}) \geq I(x_{\text{future}}; x_{\text{past}} | x_{\text{now}}) = 0$$

- Higher compression can require knowledge of the past dynamics (memories);
- Projected dynamics could differ from dynamics on the projected topology;
- $I(\cdot;\cdot)$ is Mutual Information

Put together:

$$\mathcal{F} = \alpha\, I(Y_{t+1}; Y_t, Y_{t-1}, \ldots) - I(Y_{t+1}; Y_{t-1}, \ldots | Y_t)$$

# Predictability



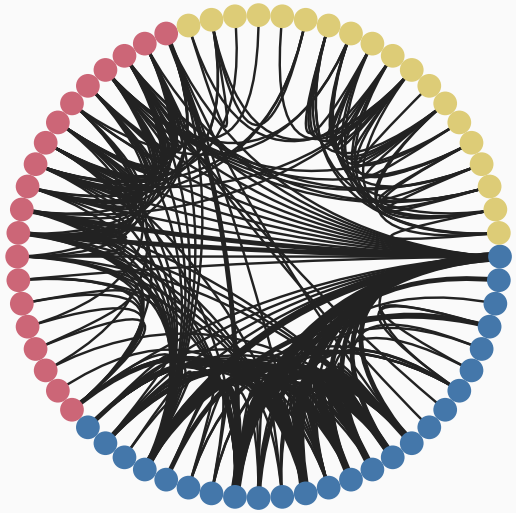
$$\mathcal{F} = \alpha\, I(Y_{t+1}; Y_t, Y_{t-1}, \ldots) - I(Y_{t+1}; Y_{t-1}, \ldots | Y_t)$$

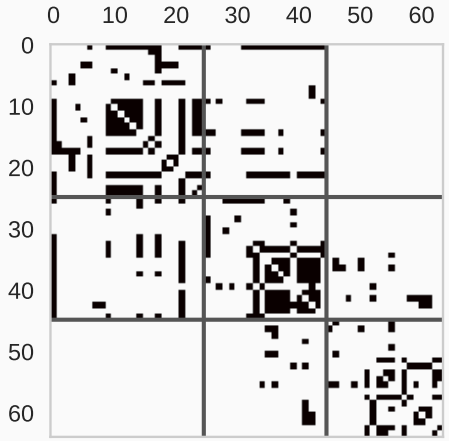
nice behavior for $\alpha = 1$:

$$\mathcal{F}_P = I(Y_{t+1}; Y_t)$$

- Easy to compute
- Under some conditions is the same as DCSBM
- Plenty of algorithms

# Blocks on Weighted Networks

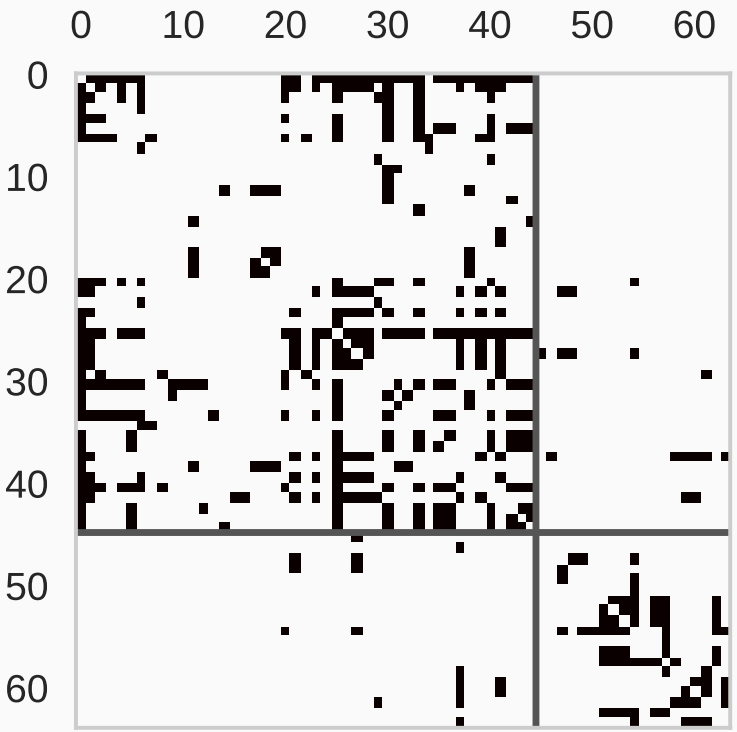





Contacts between suspect terrorists involved in the attack to Madrid station (2004).[1]

[1] *The March 11th Terrorist Network: In its weakness lies its strength, José A. Rodríguez*

# Level of Details



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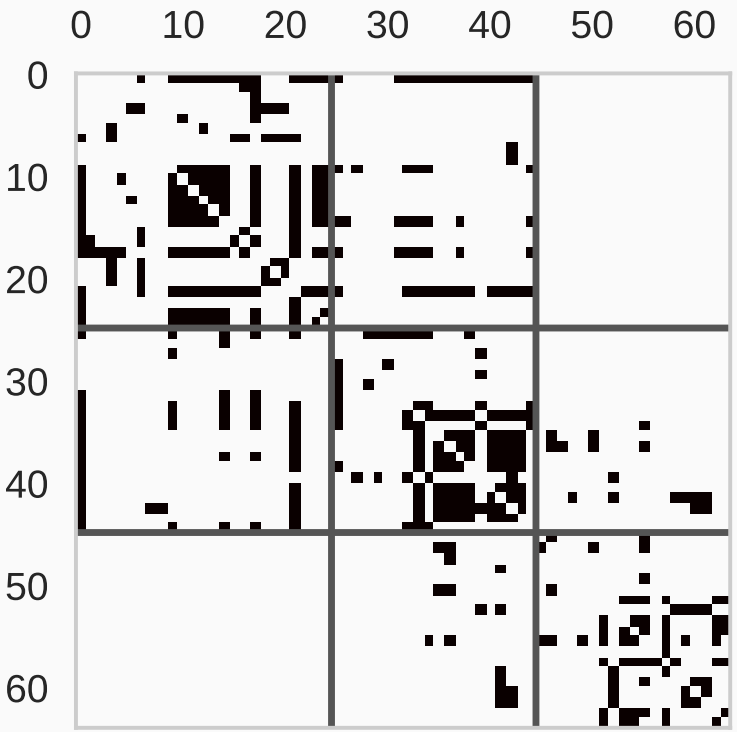

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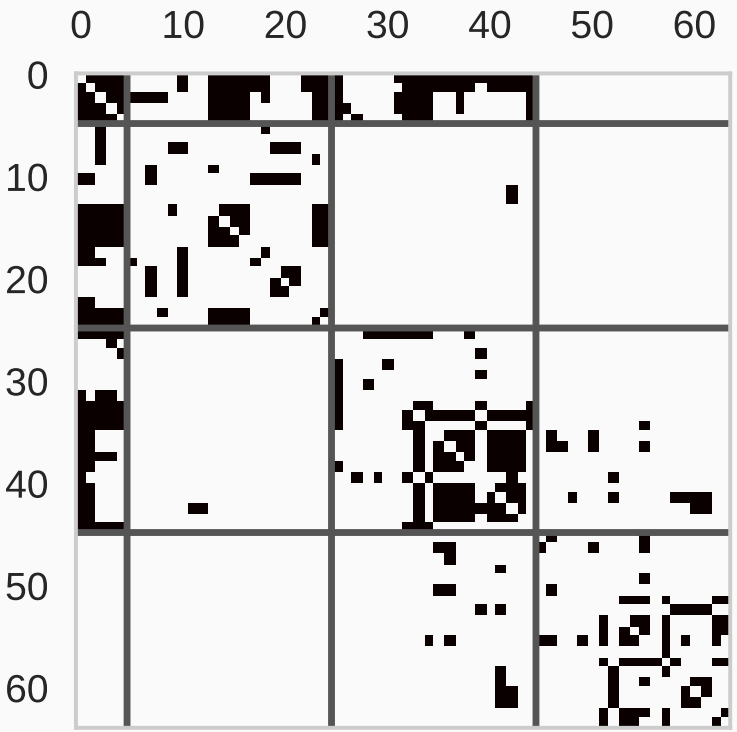

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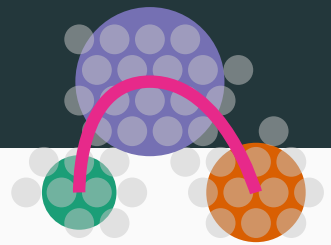

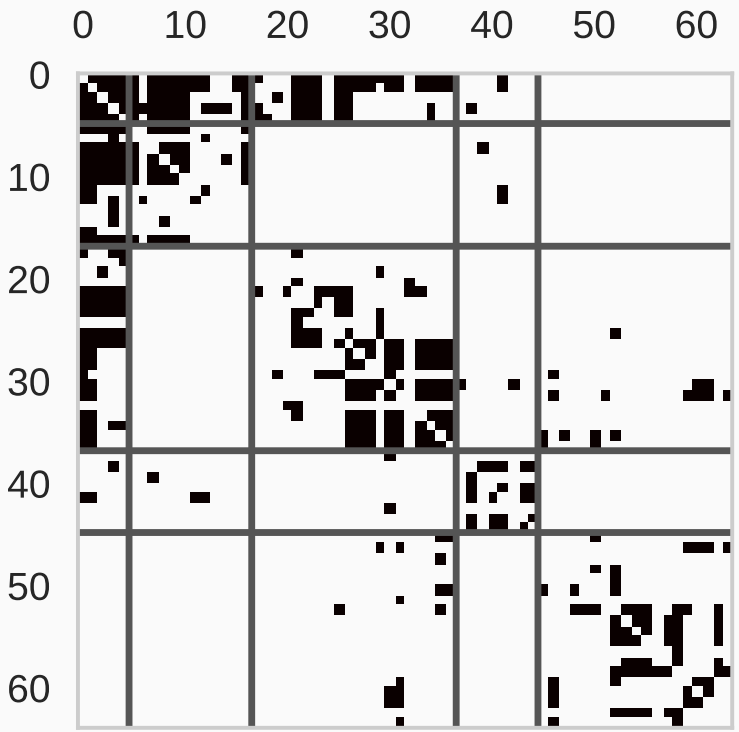

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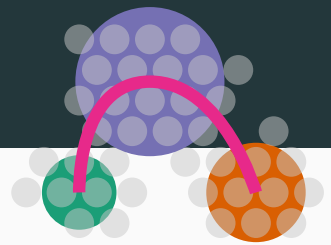
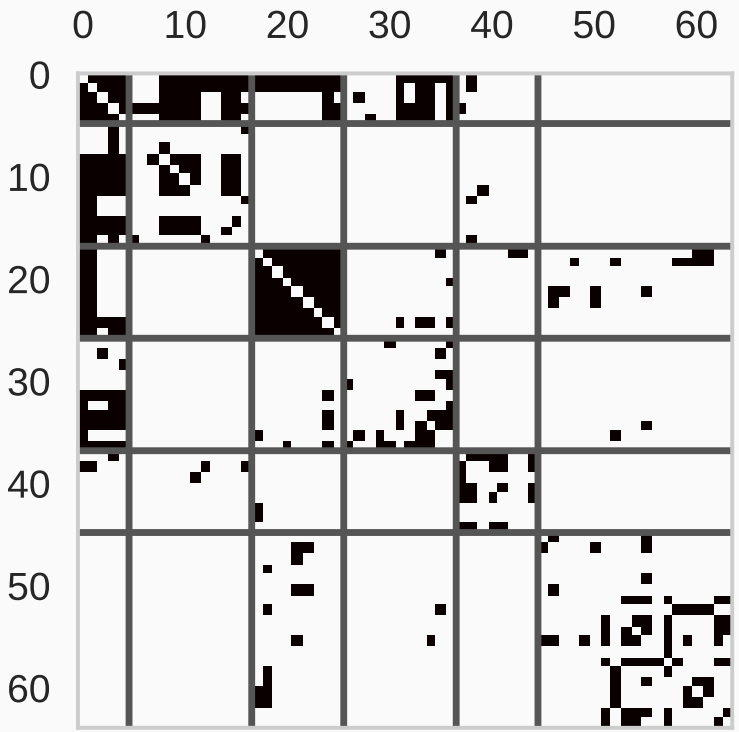
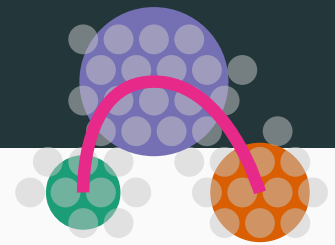




# Predictability vs Markovianity



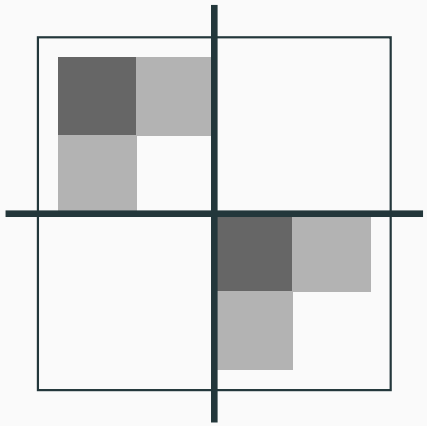
$$\mathcal{F} = \alpha\, I(Y_{t+1}; Y_t, Y_{t-1}, \ldots) - I(Y_{t+1}; Y_{t-1}, \ldots | Y_t)$$

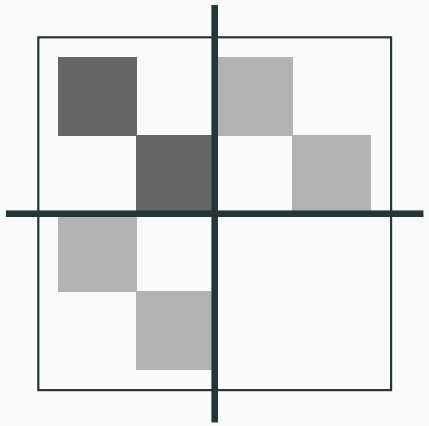
A bit harder with $\alpha = 0$:

$$\mathcal{F}_M = -I(Y_{t+1}; Y_{t-1}, \ldots | Y_t)$$

- Markovian model dynamics
- Not biased toward predictable models
- Harder to compute but easy to implement with actual algorithms

# What's the best model?



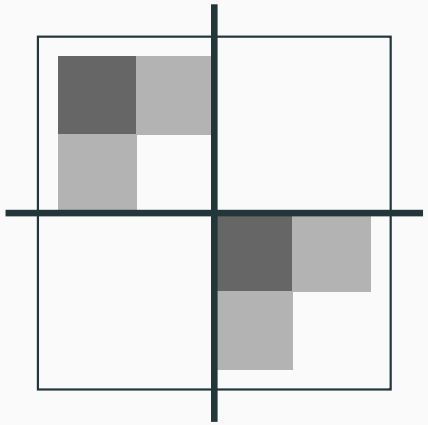


Better predictability of the dynamics

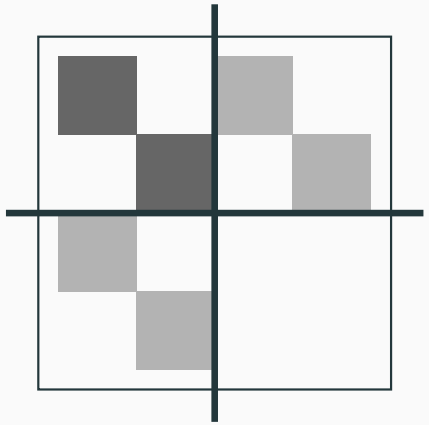
Also this is a perfect description of the system

$$\mathcal{F}_P(C) > \mathcal{F}_P(CP)$$

# What's the best model?



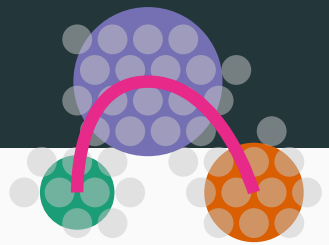
Better predictability of the dynamics



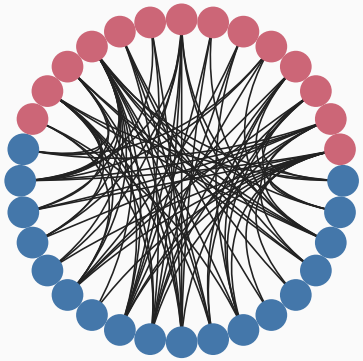
Also this is a perfect description of the system

$$\begin{aligned} \mathcal{F}_P(C) &> \mathcal{F}_P(CP) \\ \mathcal{F}_M(C) &= 0 = \mathcal{F}_M(CP) \end{aligned}$$

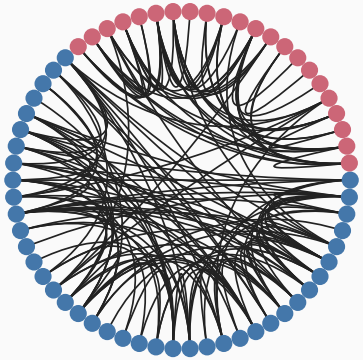
# Many others



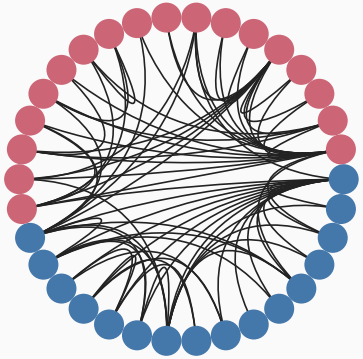
## DeepSouth



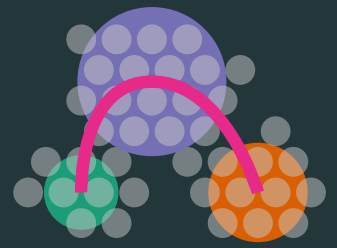

## Dolphins




## Karate

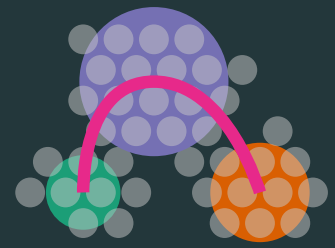


# Concluding



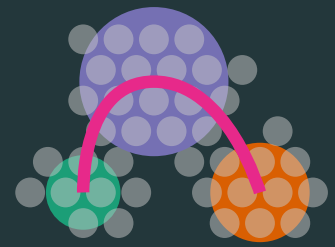
- Dynamics define partitioning

# Concluding



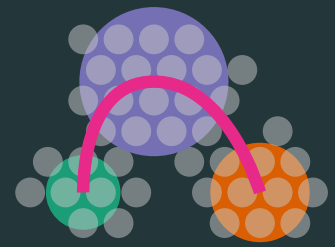
- Dynamics define partitioning
- Predictability vs Markovianity

# Concluding



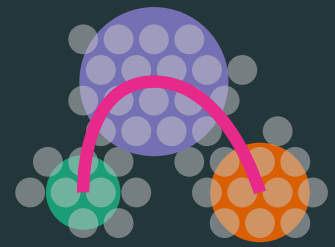
- Dynamics define partitioning
- Predictability vs Markovianity
- Only needs dynamics (topology not necessary)

# Concluding



- Dynamics define partitioning
- Predictability vs Markovianity
- Only needs dynamics (topology not necessary)
- Extends DCSBM to weighted graphs

# Concluding



- Dynamics define partitioning
- Predictability vs Markovianity
- Only needs dynamics (topology not necessary)
- Extends DCSBM to weighted graphs
- Non-Markovian Systems

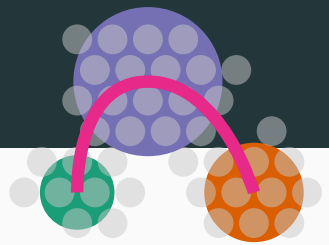
# Workshop Advertisement

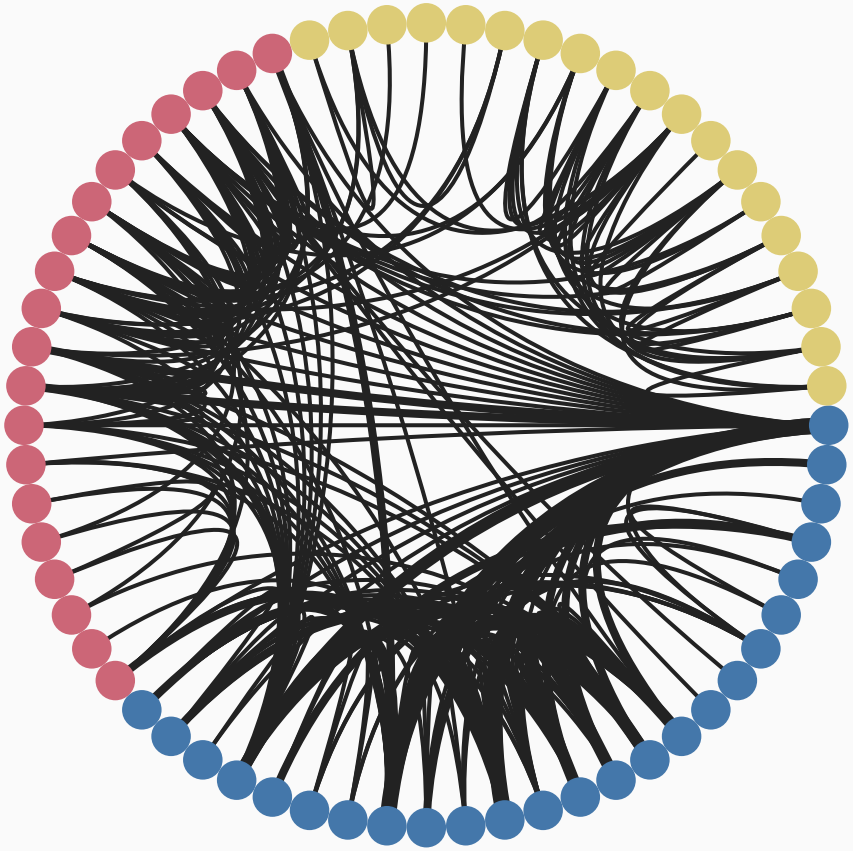
See you on Wednesday at:


Full day workshop

`http://michaelschaub.github.io/ccs_at_ccs_2016/`

# Questions?





**Joint work with:**

JC Delvenne

@ ICTEAM and BigData Group, UCLouvain.

*Good partitioning is the one leading to an interesting reduced model*