A dynamical approach to block structure analysis

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CCS 2018 – Thessaloniki

Modularity

Modularity:

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j)$$

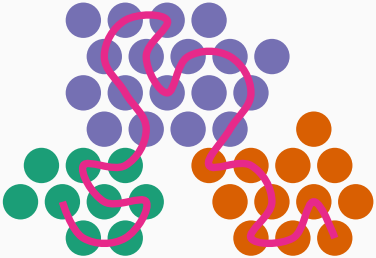
χ_c characteristic variable of partition c

$$Q \propto \sum_{c} \mathbf{Cov}\left(\chi_c(t), \chi_c(t+1)\right)$$

Projected Markov Chain

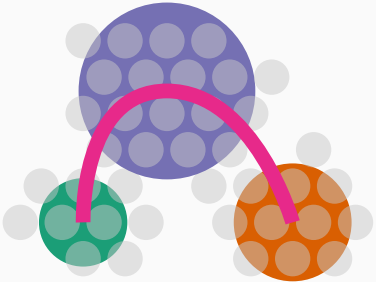
Markov Chain

$$\ldots, x_{\text{past}}, x_{\text{now}}, x_{\text{future}}, \ldots$$

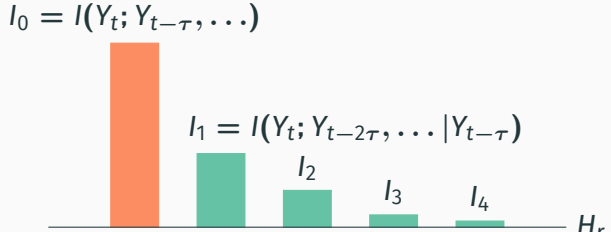


Projection

$$\ldots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \ldots$$



Entrogram



where $I(X;Y) = H(X) - H(X|Y)$ is the Mutual Information

M.F. et al, Journal of Complex Networks, cnx055

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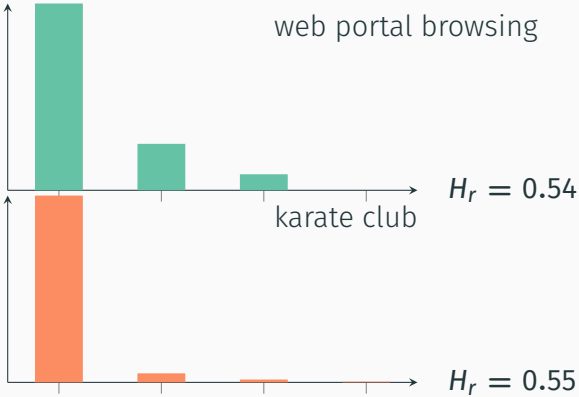
Entrogram: Examples



Non linear communities

Modularity

$$Q \propto \sum_c \mathbf{Cov}\left(\chi_c(t), \chi_c(t+1)\right)$$

Objective function: $I(Y_t, Y_{t-\tau})$

DCSBM

$$I(Y_t; Y_{t-\tau}) \propto -\sum_{rs} e_{rs} \log \frac{e_{rs}}{e_r e_s}$$

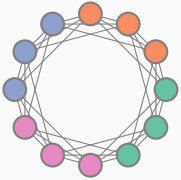
In some cases

Examples

Example 1: One cycle

How many Partitions?

Adj:



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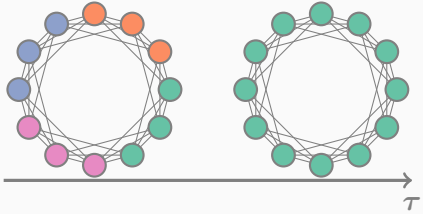
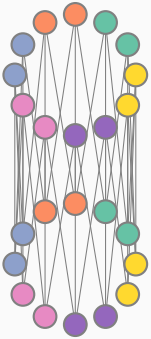


Example 2: Two cycles



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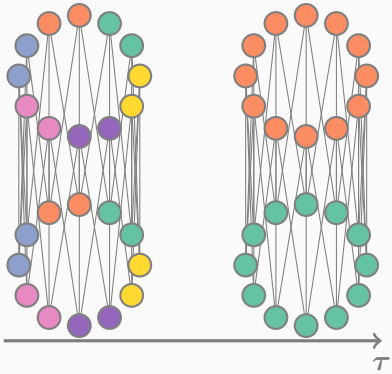

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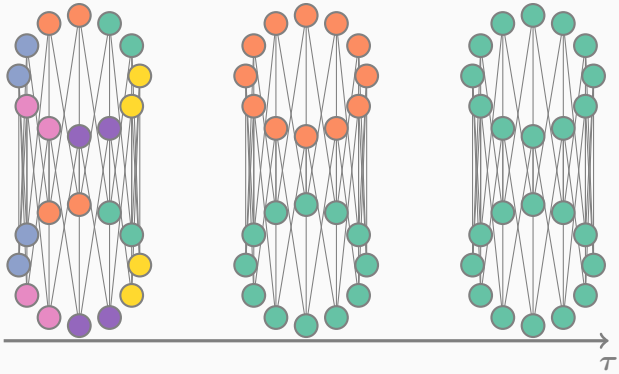
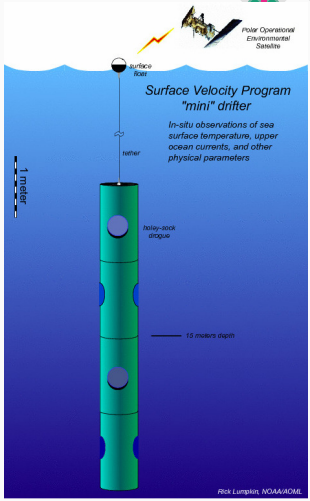
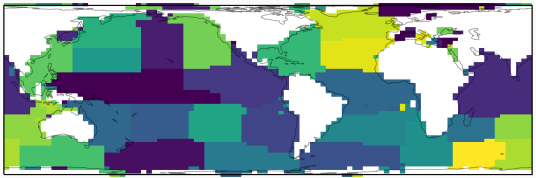


Example 3. Ocean buoys



Example 3. Ocean buoys

$\tau = 7$ days



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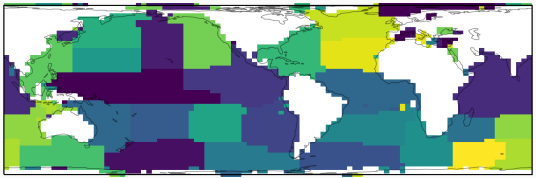
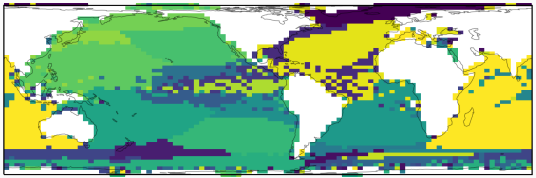


$\tau = 700$ days



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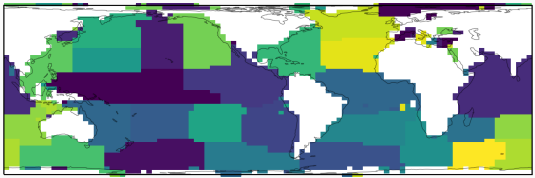

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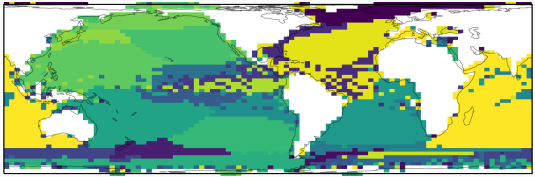

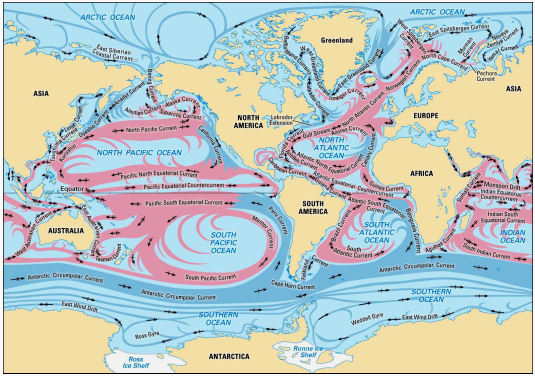

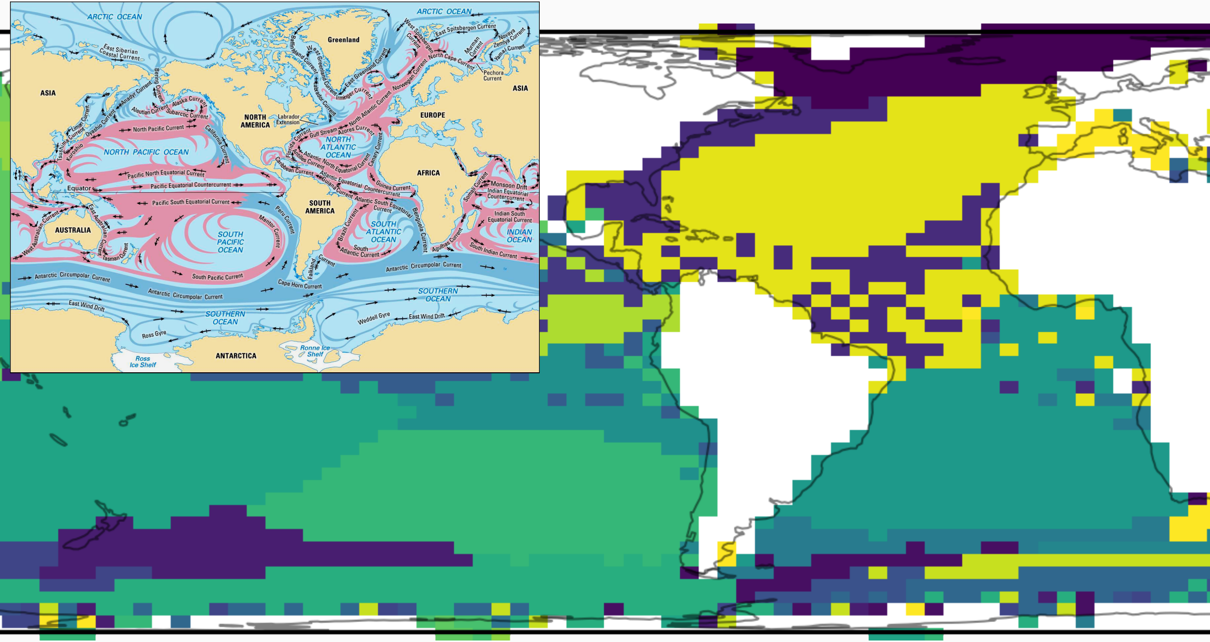
ARCTIC OCEAN
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East Siberian Coastal Current
Greenland
ASIA
ASIA
Bering Current
Anadyr Current
Alaska Current
Aleutian Current
Subarctic Current
North Pacific Current
NORTH PACIFIC OCEAN
California Current
Kuroshio
Oyashio Current
Tsushima Current
Liman Current
NORTH AMERICA
Labrador Extension
Gulf Stream
Azores Current
North Atlantic Current
Norwegian Current
North Cape Current
East Spitsbergen Current
West Spitsbergen Current
Murman Current
Novaya Zemlya Current
Yamal Current
Pechora Current
Irminger Current
East Greenland Current
West Greenland Current
Baffin Island Current
Labrador Current
Florida Current
Antilles Current
Caribbean Current
NORTH ATLANTIC OCEAN
Atlantic North Equatorial Current
Canary Current
EUROPE
AFRICA
Guinea Current
Atlantic Equatorial Countercurrent
Guiana Current
Atlantic South Equatorial Current
Pacific North Equatorial Current
Pacific Equatorial Countercurrent
Equator
Pacific South Equatorial Current
SOUTH AMERICA
Peru Current
Mentor Current
Brazil Current
Benguela Current
SOUTH ATLANTIC OCEAN
South Atlantic Current
Agulhas Current
Somali Current
Monsoon Drift
Indian Equatorial Countercurrent
Indian South Equatorial Current
INDIAN OCEAN
South Indian Current
AUSTRALIA
East Australian Current
West Australian Current
Tasman Current
SOUTH PACIFIC OCEAN
South Pacific Current
Falkland Current
Cape Horn Current
Antarctic Circumpolar Current
Antarctic Circumpolar Current
Antarctic Circumpolar Current
East Wind Drift
East Wind Drift
SOUTHERN OCEAN
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Weddell Gyre
Ross Gyre
Ronne Ice Shelf
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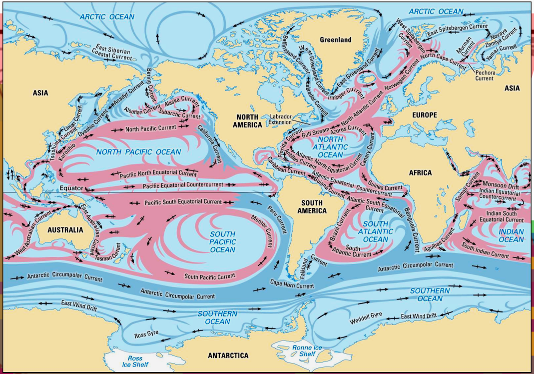
ANTARCTICA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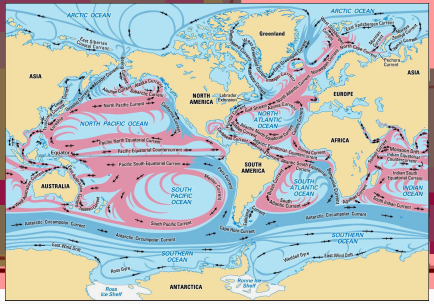
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Concluding

- A information theoretical algorithm for block detection
- As a plus: same base as (DC)SBM (dynamical interpretion)
- Weighted networks and non-networks (only trajectories)
- Code at: `https://github.com/maurofaccin/entropart`

Questions?

FIG. 172.—FRANKLIN'S CHART OF THE GULF STREAM.

Joint work with:

JC Delvenne — icteam

M Schaub — IDSS

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