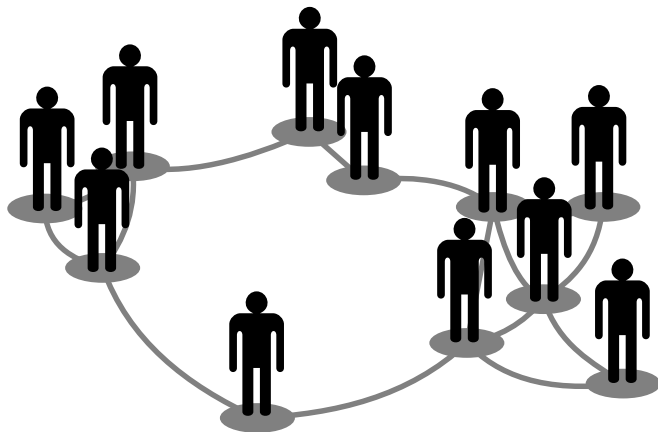# Community Detection in Quantum Systems
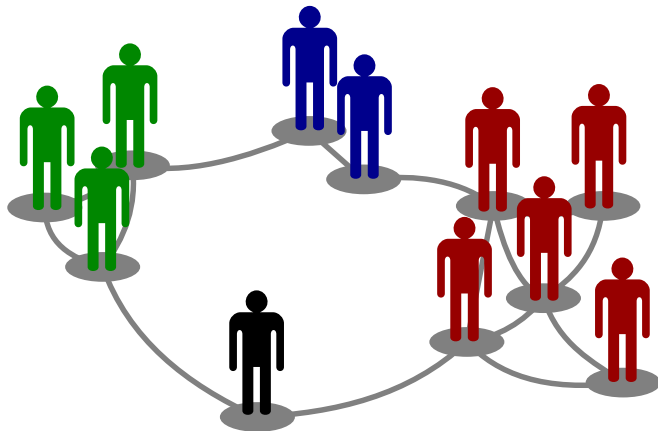
## Mauro Faccin
### ISI Foundation

CompleNet2014
Bologna, March 12-14, 2014
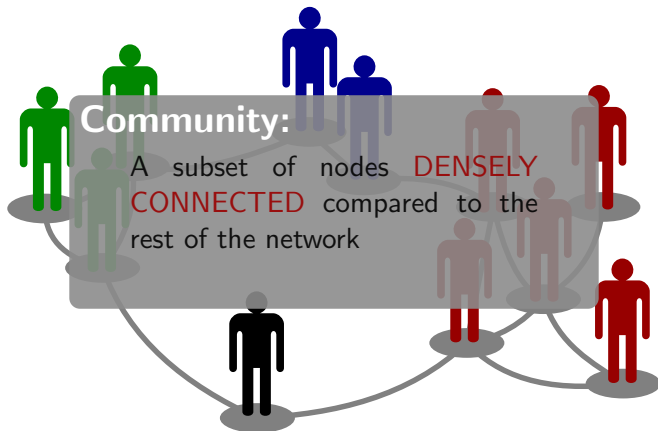
# Communities: an overview

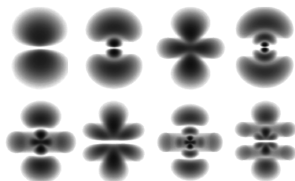# Communities: an overview

Communities: an overview



**Community:**
A subset of nodes DENSELY CONNECTED compared to the rest of the network

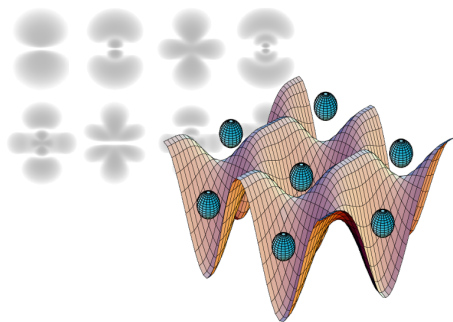Quantum Systems: an overview

# Quantum Systems: an overview



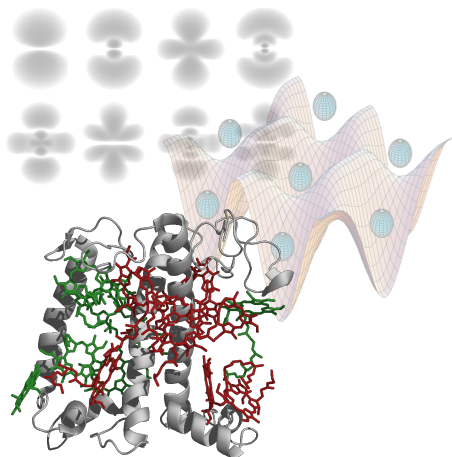► Simple Systems
  (Atomic Orbitals. . . )

# Quantum Systems: an overview



- ▶ Simple Systems
  (Atomic Orbitals. . . )
- ▶ Symmetric Systems
  (Lattices. . . )

# Quantum Systems: an overview



- ▶ Simple Systems
  (Atomic Orbitals...)
- ▶ Symmetric Systems
  (Lattices...)
- ▶ Complex Systems
  (Proteins...)

# Quantum Mechanics & Complex Networks

- ▶ Quantumness on quantum walks (relation with degree distribution)
- ▶ Chiral walks (new effects in quantum walks)
- ▶ Quantum internet (Quantum Google, . . . )

# Community Detection, the Usual Way

- ▶ Network characterization (eigenvalues, edges ranking. . . )
- ▶ Statistical significance
- ▶ Random walker: stay trapped in a community
- ▶ . . .

**Available methods:**

Girvan and Newman, Modularity optimization, Spin models, Clique percolation, Spectral methods, OSLOM, InfoMap, COPrA, Stability . . .

# What's a Quantum Community?

A quantum community should show "quantum properties" such as:

- ▶ Interference
- ▶ Coherence
- ▶ Entanglement
- ▶ . . .

### Our approach:

- ▶ Quantum walks on a graph (one particle subspace)
- ▶ Define a closeness measure between nodes (should include *quantum behavior*)
- ▶ Based on node closeness find communities (hierarchical clustering, modularity maximization)

## Quantum Community: Transport Approach

Transfer matrix:

$$T_{ij}(t) = |\langle i|e^{-iHt}|j\rangle|^2$$

Minimize the probability that a quantum walker leave a community

**Closeness:**

$$c_t^T(A, B) = \frac{1}{|A||B|} \sum_{i\in A, j\in B} T_{ij}(t) + T_{ji}(t)$$

Two communities $A$ and $B$ are close if the flow between them is high.

# Quantum Community: Fidelity Approach

Fidelity:

$$F(\rho, \sigma) = |\langle \psi | \phi \rangle|$$

for pure states:

$$\rho = |\psi\rangle \langle \psi|$$
$$\sigma = |\phi\rangle \langle \phi|$$

Maximize the coherece between nodes within a community.

**Closeness:**

$$c_t^F(A, B) = \frac{\mathcal{F}_A(t) + \mathcal{F}_B(t) - \mathcal{F}_{A \cup B}(t)}{|A||B|}$$

where

$$\mathcal{F}_X(t) = F^2(\rho_A(t), \rho_A(0))$$
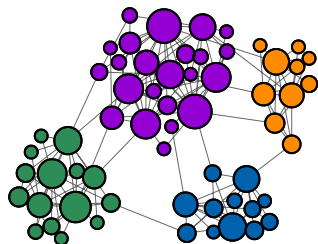
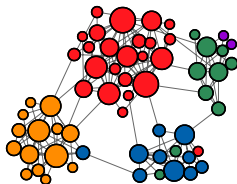Two communities $A$ and $B$ are close if the coherence between them persists.

# LFR Example

Benchmark network:
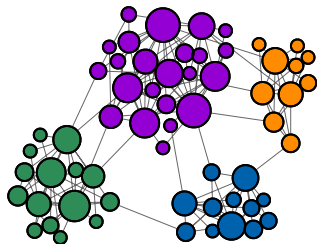


Lancichinetti et al., PRE, 78:046110, 2008.

# LFR Example

Benchmark network:
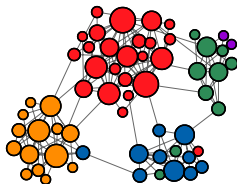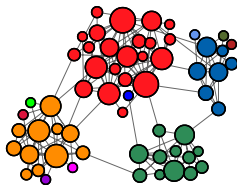
Transport

Fidelity
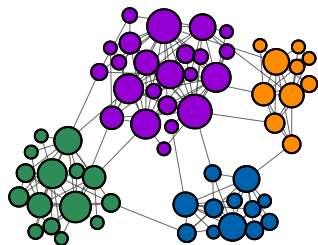


Lancichinetti et al., PRE, 78:046110, 2008.

# LFR Example



Transport

Transport
(short time)

Benchmark network:

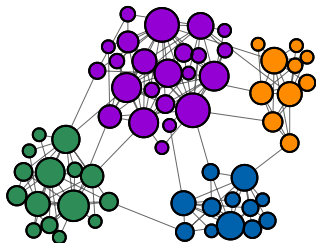Fidelity

OSLOM

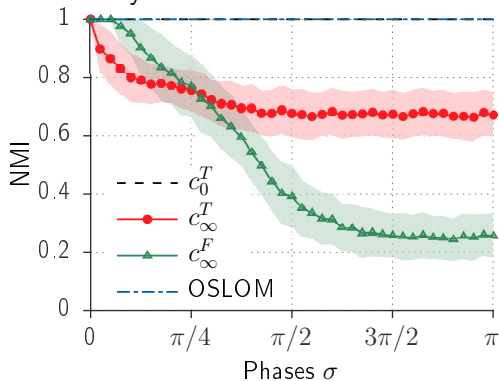Lancichinetti et al., PRE, 78:046110, 2008.
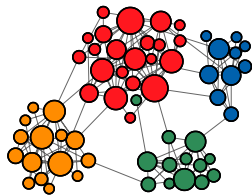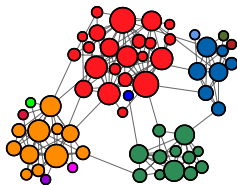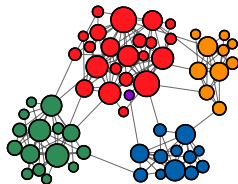
# LFR Example — Phases

Benchmark network:



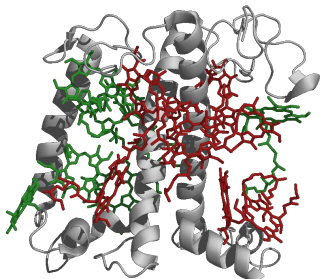Lancichinetti et al., PRE, 78:046110, 2008.

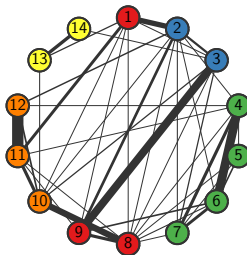Phases induce changes in the community structure:



In presence of complex hopping terms, the system dynamics changes drastically (Magnetic fields, effective Hamiltonian).
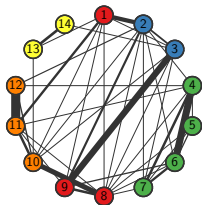
# LHCII

Light harvesting systems LHCII:

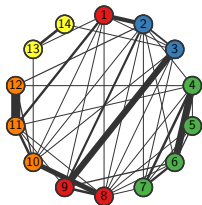14 nodes network with Hamiltonian $H$.

# LHCII

Light harvesting systems
LHCII:



Community structure from
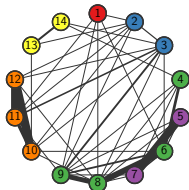literature.
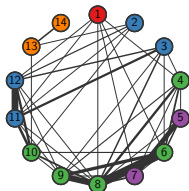
# LHCII

Light harvesting systems
LHCII:



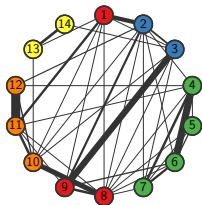Community structure from
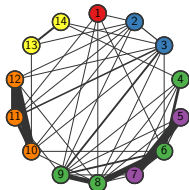literature.

Transport



Fidelity

# LHCII

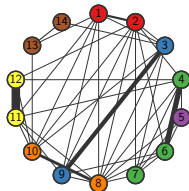Light harvesting systems
LHCII:



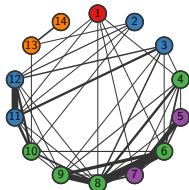Community structure from
literature.

### Transport



### Fidelity



Transport
(short time)



Semi classical
approach.

# Comments

- ▶ A community detection algorithm based on quantum mechanics
- ▶ Community detection FOR quantum systems
- ▶ Quantum effects $\rightarrow$ closeness measures
- ▶ Consistent community structures for LCHII

Ads: arXiv:1310.6638
with Piotr Migdał, Tomi Johnson, Ville Bergholm
and Jacob Biamonte

Mauro.Faccin@isi.it                    www.TheQuantumNetwork.org