Beyond Communities: Dynamics Define Modules

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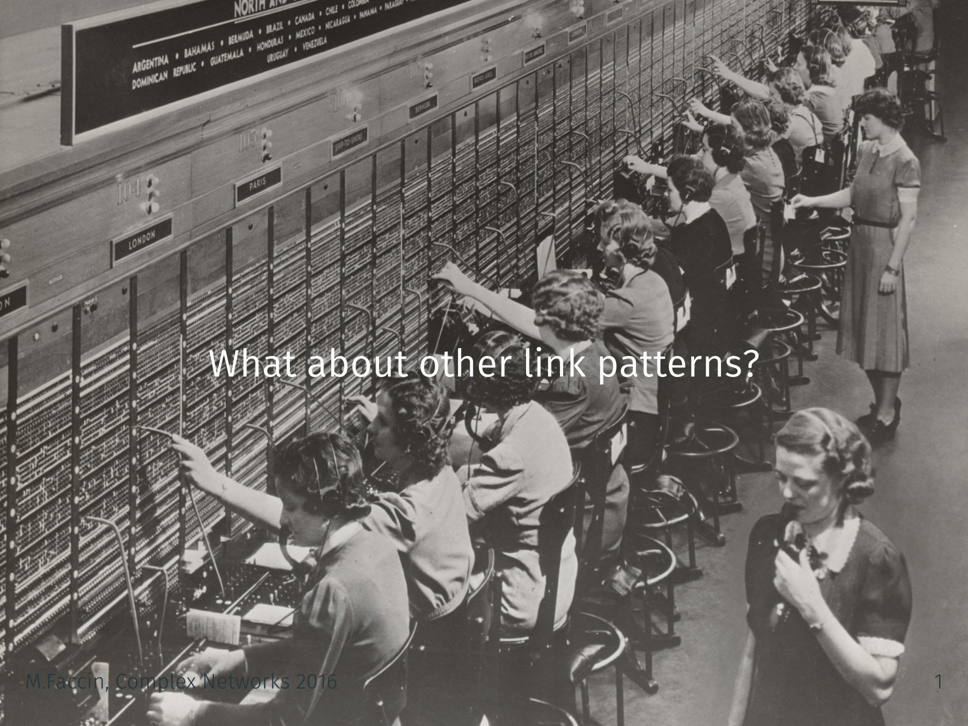
Complex Networks 2106 – Milan



What about other link patterns?

Modules defined by dynamics

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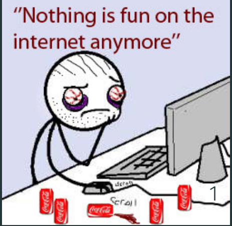
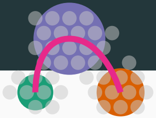


Projected Markov Chain

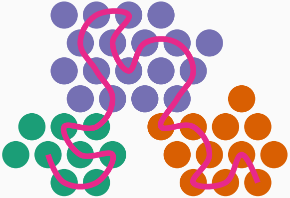


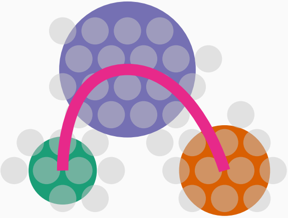
Markov Chain

$$\ldots, x_{\text{past}}, x_{\text{now}}, x_{\text{future}}, \ldots$$

Projection

$$\ldots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \ldots$$





Complexity

Complexity

Memories

Projected Markov Chain:

$$\ldots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \ldots$$

Maximize predictability:

$$I(Y_{\text{future}}; Y_{\text{past}})$$

amount of information flowing from *past* to *future*.

$$I(Y_{\text{future}}; Y_{\text{past}}) \leq I(x_{\text{future}}; x_{\text{past}})$$

- Increases predictability of future with knowledge of past;
- Favors heterogeneous module-linking;
- Favors homogeneous module size;
- $I(\cdot;\cdot)$ is Mutual Information

Assess the Partition Quality 2

Projected Markov Chain:

$$\dots, Y_{\text{past}}, Y_{\text{now}}, Y_{\text{future}}, \dots$$

Minimize memories:

$$I(Y_{\text{future}}; Y_{\text{past}} | Y_{\text{now}})$$

higher order memory embedded into the process.

$$I(Y_{\text{future}}; Y_{\text{past}} | Y_{\text{now}}) \geq I(x_{\text{future}}; x_{\text{past}} | x_{\text{now}}) = 0$$

- Higher compression can require knowledge of the past dynamics (memories);
- Projected dynamics could differ from dynamics on the projected topology;
- $I(\cdot;\cdot)$ is Mutual Information

Put together:

$$\mathcal{F} = \alpha\, I(Y_{t+1}; Y_t, Y_{t-1}, \ldots) - I(Y_{t+1}; Y_{t-1}, \ldots | Y_t)$$

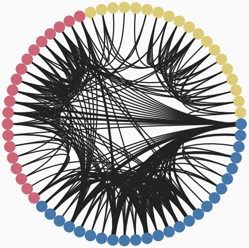
Predictability

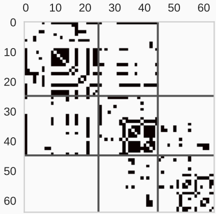
$$\mathcal{F} = \alpha\, I(Y_{t+1}; Y_t, Y_{t-1}, \ldots) - I(Y_{t+1}; Y_{t-1}, \ldots \,|Y_t)$$

nice behavior for $\alpha = 1$:

$$\boxed{\mathcal{F}_P = I(Y_{t+1}; Y_t)}$$

- Easy to compute
- Under some conditions is the same as DCSBM
- Plenty of algorithms

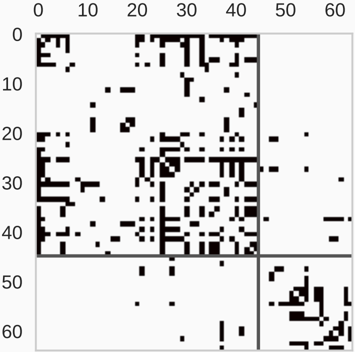


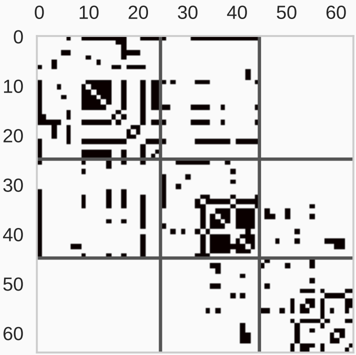


Contacts between suspect terrorists involved in the attack to Madrid station (2004).[1]

[1] *The March 11th Terrorist Network: In its weakness lies its strength, José A. Rodríguez*

Level of Details

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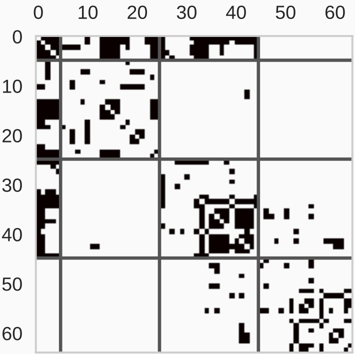
Level of Details



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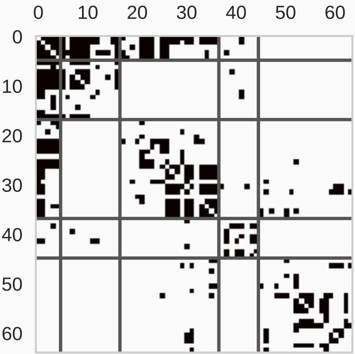

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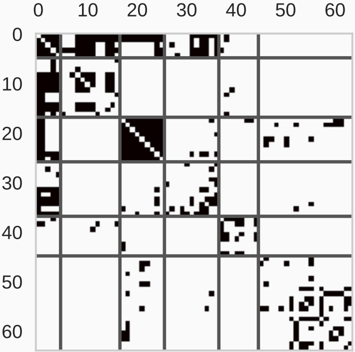
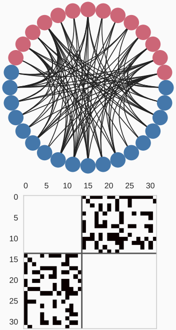
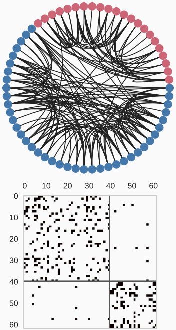


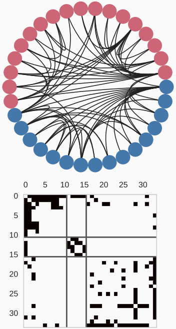
DeepSouth



Dolphins



Karate



Predictability vs Markovianity

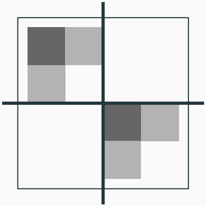
$$\mathcal{F} = \alpha\, I(Y_{t+1}; Y_t, Y_{t-1}, \ldots) - I(Y_{t+1}; Y_{t-1}, \ldots | Y_t)$$

A bit harder with $\alpha = 0$:

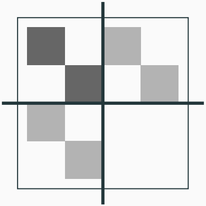
$$\boxed{\mathcal{F}_M = -I(Y_{t+1}; Y_{t-1}, \ldots | Y_t)}$$

- Markovian model dynamics
- Not biased toward predictable models
- Harder to compute but easy to implement with actual algorithms

What’s the best model?



Better predictability of the dynamics



Also this is a perfect description of the system

$$\mathcal{F}_P(C) \quad > \quad \mathcal{F}_P(CP)$$

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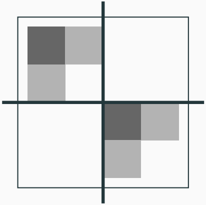

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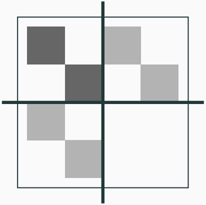

Also this is a perfect description of the system

$$
\begin{aligned}
\mathcal{F}_P(C) &\quad > \quad \mathcal{F}_P(CP) \\
\mathcal{F}_M(C) &= 0 = \mathcal{F}_M(CP)
\end{aligned}
$$

Non Markovian Dynamics

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One can image a the Erdős–Rényi city with two quartiers where:

- people from each quartiers go to work on the morning;
- they go back home at night

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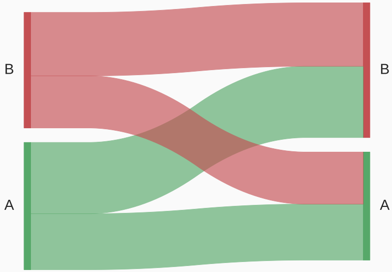
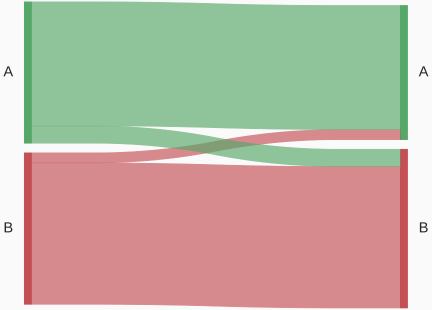
$$I(Y_t|Y_{t-1}, Y_{t-2})$$

$I(Y_t; Y_{t-1})$
without memories:
no structure



Non Markovian Dynamics

$I(Y_t; Y_{t-1}, Y_{t-2})$
considering memories: structure in dynamics



Concluding

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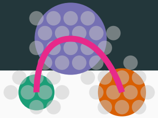
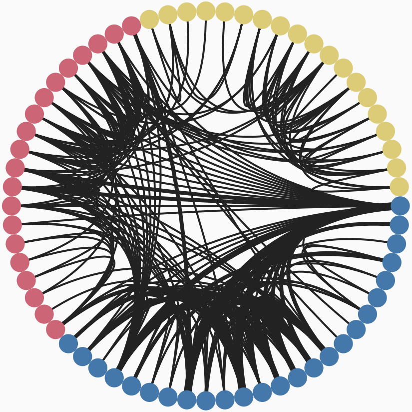
- Dynamics define partitioning
- Predictability vs Markovianity
- Only needs dynamics (topology not necessary)
- Extends DCSBM to weighted graphs
- Non-Markovian Systems

Questions?





Joint work with:

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`https://maurofaccin.github.io`

Good partitioning is the one leading to an interesting reduced model