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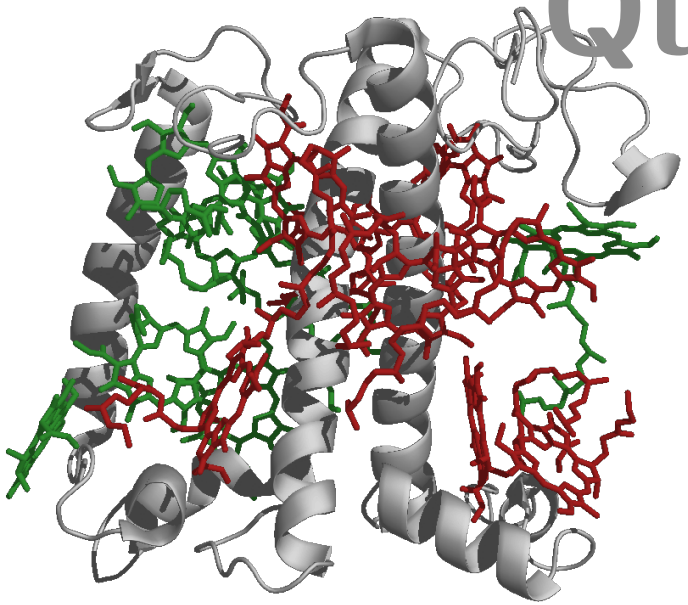

Quantum Approaches to Community Detection

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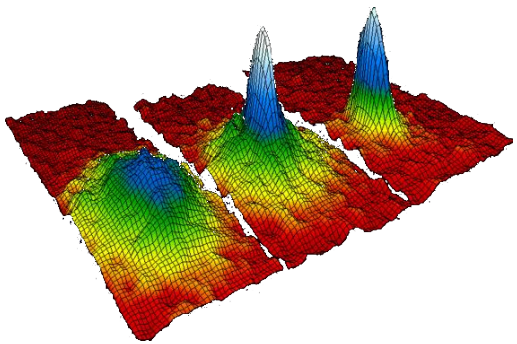
ECCS2014

Lucca, September 23, 2014

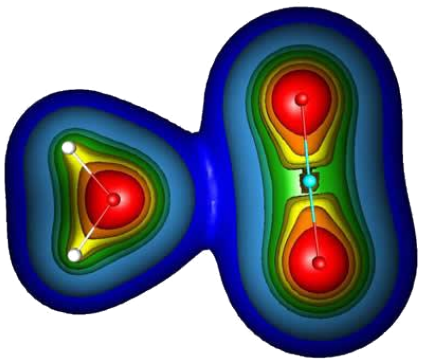
Quantum Systems: tractable structures

No structure



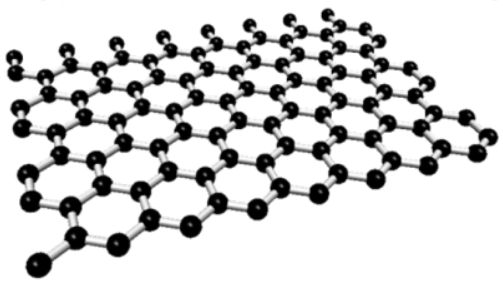
(Fermion/boson gas)

Small Structure



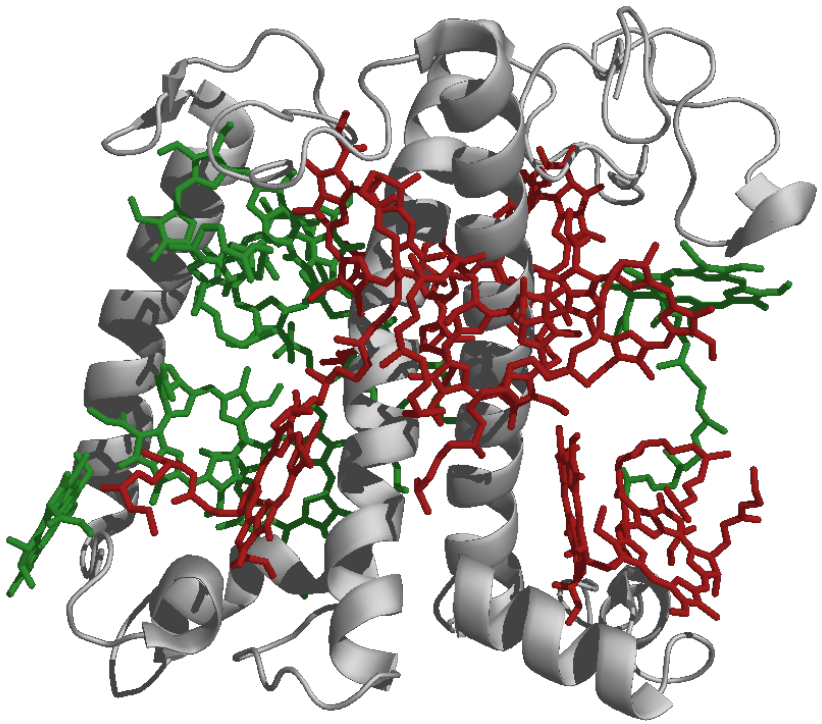
(Few atoms)

Periodic Structure



(Solid state)

Quantum Systems: complex structure



Complex Structure

Cases:

- Energy transport
- Quantum communication

Goals of the approach:

- Simulation
- Design

Communities! Quantum or Classical?

Learn from Classical

- Systematic approach
- Classical techniques
- Difficulties

Differences

- System size
- Dynamics
- Algorithm efficiency not fundamental

The Task

Parts:

$\mathcal{H}$ a Hilbert space

$\{|i\rangle\}$ network nodes represent the orthonormal basis

H the Hamiltonian defines the system dynamics

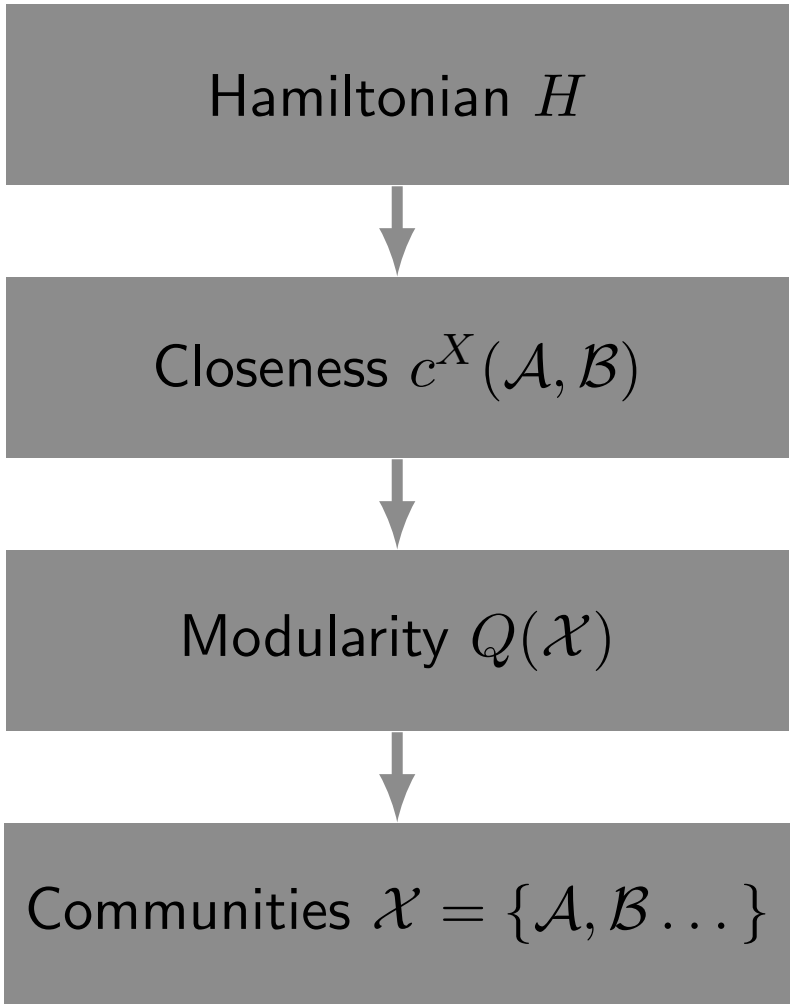
Picture:

Find a community structure $\mathcal{X} = \{\mathcal{A}, \mathcal{B}, ..\}$ such that:

$$\mathcal{V}_{\mathcal{A}} = \operatorname*{span}_{i \in \mathcal{A}} \{|i\rangle\}$$

$$\mathcal{H} = \bigoplus_{\mathcal{A} \in \mathcal{X}} \mathcal{V}_{\mathcal{A}}$$

Our Approach



determines system dynamics

we decide a physical quantity X
we want to be small or big

minimize that quantity

output partition

Quantum Community: Transport Approach

Transfer matrix:

$$T_{ij}(t) = |\langle i|\mathrm{e}^{-\mathrm{i}Ht}|j\rangle|^2$$

Minimize the probability that a quantum walker leave a community

Closeness:

$$c_t^T(A,B) = \frac{1}{|A||B|}\sum_{i\in A, j\in B} T_{ij}(t)+T_{ji}(t)$$

Two communities A and B are close if the transport flow between them is high.

Quantum Community: Fidelity Approach

Fidelity:

$$F(\rho, \sigma) = |\langle\psi|\phi\rangle|$$

for pure states:

$$\rho = |\psi\rangle\langle\psi|$$
$$\sigma = |\phi\rangle\langle\phi|$$

Maximize the coherence between nodes within a community.

Closeness:

$$c_t^F(A, B) = \frac{\mathcal{F}_A(t) + \mathcal{F}_B(t) - \mathcal{F}_{A\cup B}(t)}{|A||B|}$$

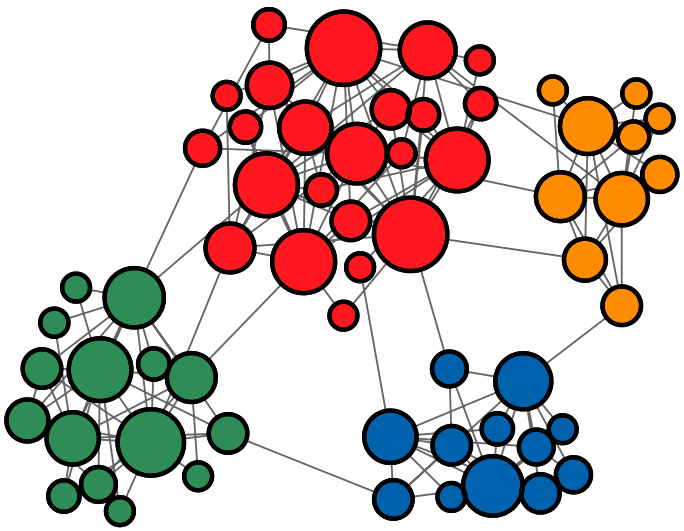
where

$$\mathcal{F}_X(t) = F^2(\rho_A(t), \rho_A(0))$$

Two communities A and B are close if the coherence between them persists.

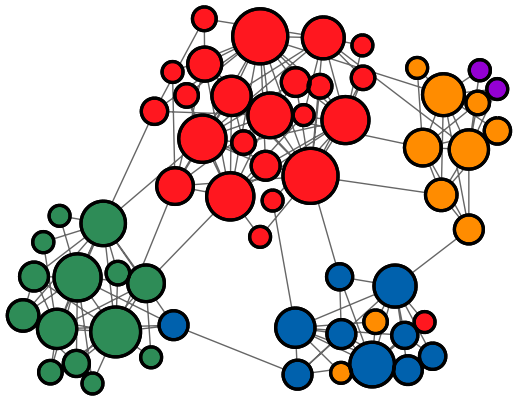
An Example

Benchmark network:

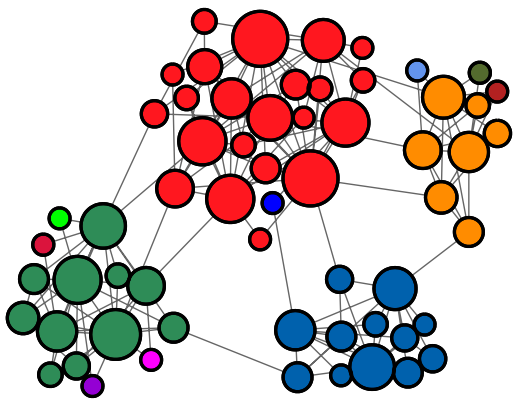


Lancichinetti et al., PRE, 78:046110, 2008.

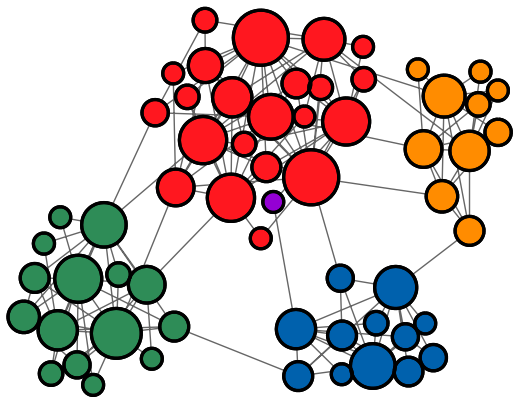
Transport



Fidelity

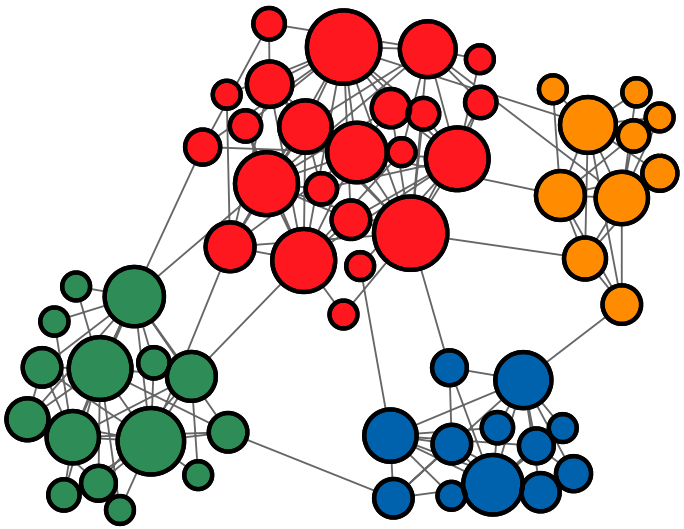


OSLOM



An Example

Benchmark network:



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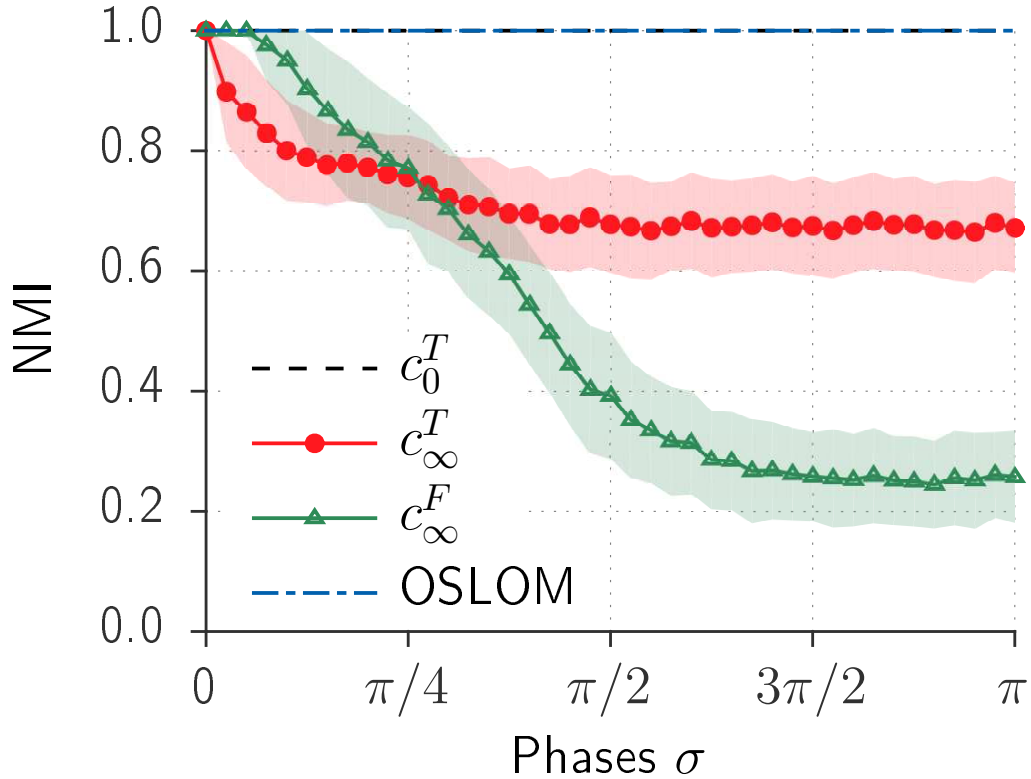
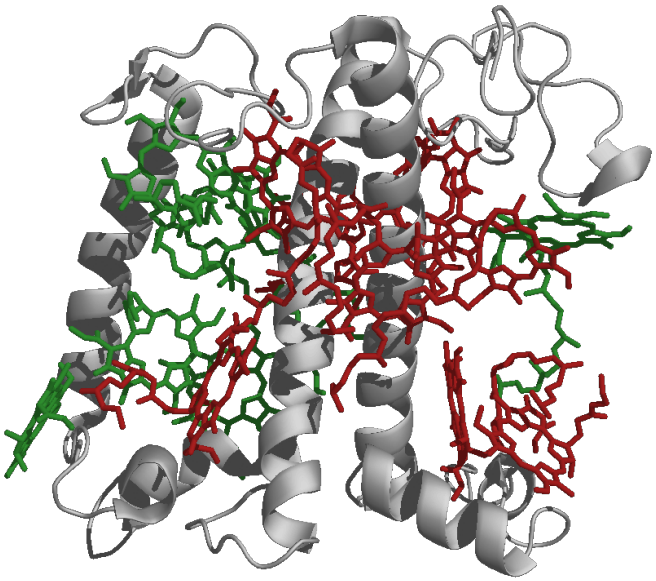
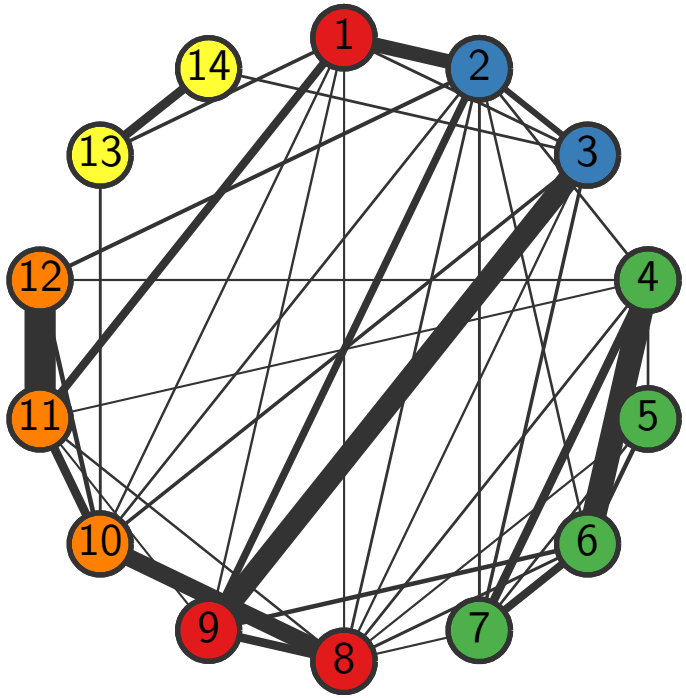


LHCII

Light harvesting systems LHCII:

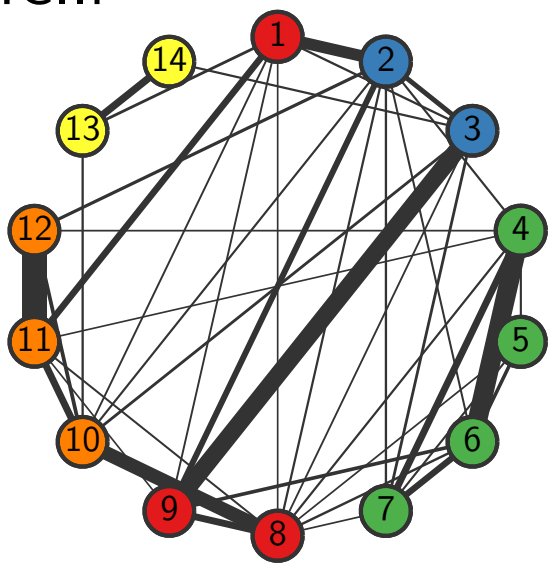


14 nodes network with Hamiltonian H.

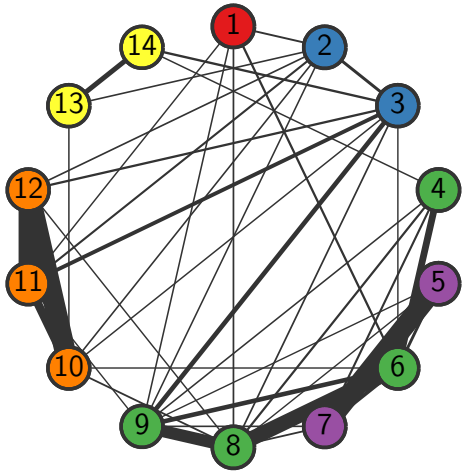


LHCII

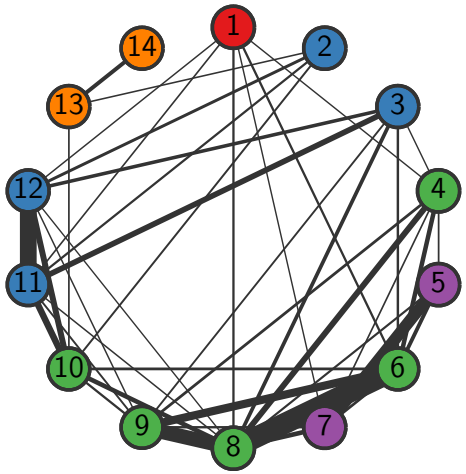
Light harvesting systems LHCII:



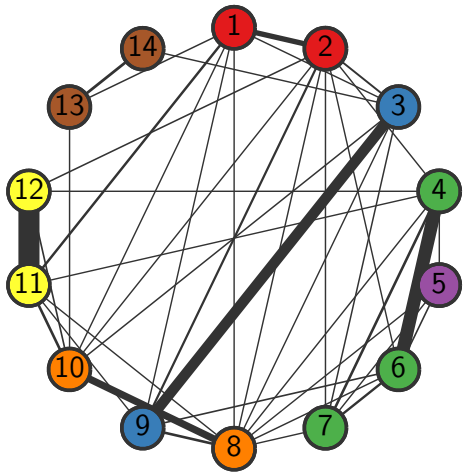
Transport



Fidelity



Only interaction strength



(classical)

Conclusion:

- A community detection algorithm based on quantum mechanics
- Community detection FOR quantum systems
- Classical methods are not entirely appropriate
- More possibilities should depends on specific applications

Any Question/Comment?

Community Detection in Quantum Complex Networks

arXiv:1310.6638

Now accepted in Phys.Rev.X for publication

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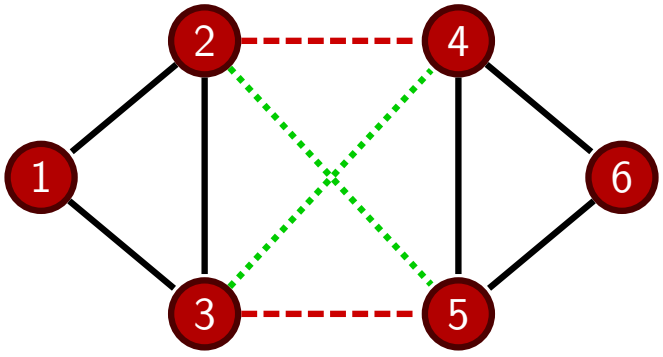
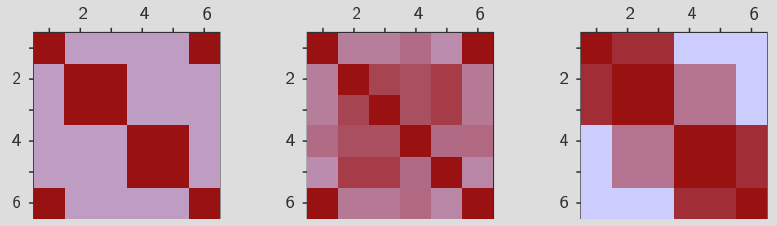
Model for Quantum Systems



Toy graph to reveal non-intuitive *quantum effects*

Closeness

Phases' effect on transport:



All zeros | Random | Canceling