Degree Distribution in Quantum Complex Networks

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IQC workshop on quantum computation
and complex networks

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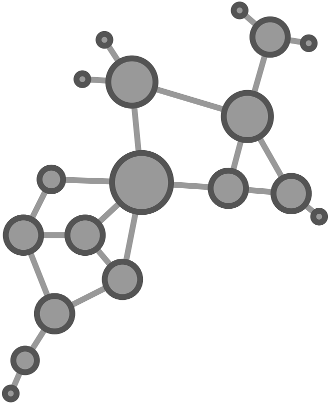
Institute for Scientific Interchange
Quantum Science Lab

Outline

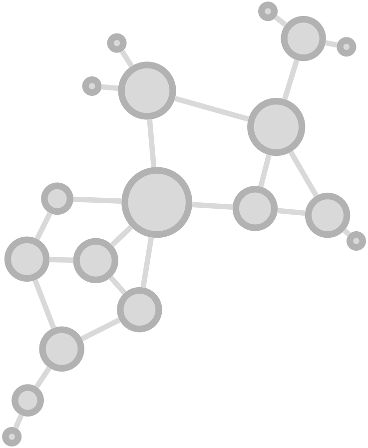
- Short intro to degree distribution (stochastic random walk)
- Quantum generator and Probability Distribution (quantum *steady state*)
- Quantum Correction to the Classical Behavior
- Entropy Bound

The Graph



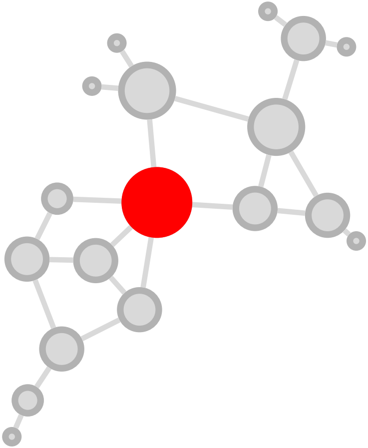
- A : Adjacency matrix
- D : Matrix with node degrees on the diagonal
- $\mathcal{L}$: Laplacian matrix

Random Walks



- start from a node
- choose a neighbor
- move to it

Random Walks



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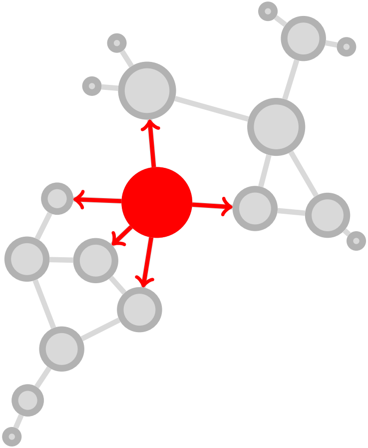
Random Walks



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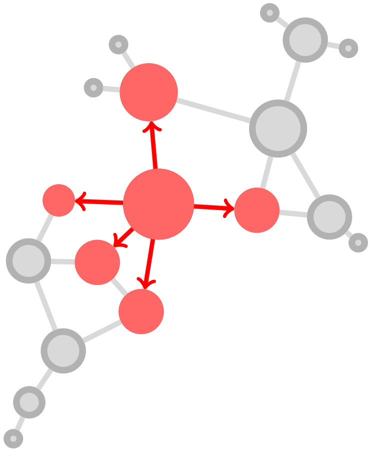
Random Walks



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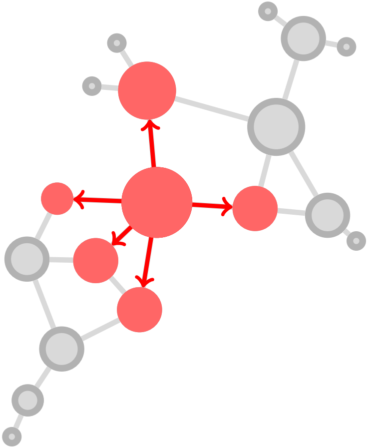
Random Walks



- start from a node
- choose a neighbor
- move to it

Transition Matrix:

$$\begin{pmatrix} & & & & & \vdots & & & & \\ & & & & & 1/5 & & & & \\ & & & & & 1/5 & & & & \\ & & & & & 1/5 & & & & \\ & & & & & \vdots & & & & \\ \dots & 1/3 & 1/3 & 1/2 & \dots & 0 & \dots & 1/3 & 1/4 & \dots \\ & & & & & \vdots & & & & \\ & & & & & 1/5 & & & & \\ & & & & & 1/5 & & & & \\ & & & & & \vdots & & & & \end{pmatrix} = AD^{-1}$$

Degree and Stochastic Processes

Stochastic Generator:

$$H_C = \mathcal{L}D^{-1}$$

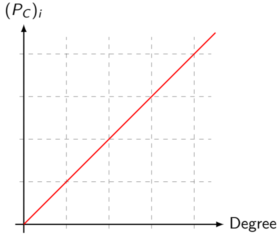
Probability distribution at time t:

$$P_C(t) = \mathrm{e}^{H_C t} P_C(0)$$

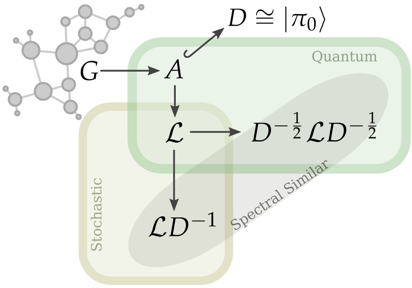
The eigenvector with zero eigenvalue is:

$$\bar{\phi}_0 = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix}$$

Linear correlation between *degree* and probability distribution at the *steady state* of the stochastic process.



Quantum Generator



- initial state
- long time average

Quantum Generator

$$H_Q = D^{-\frac{1}{2}}\mathcal{L}D^{-\frac{1}{2}}$$

with the same spectrum of $\mathcal{L}D^{-1}$.
The probability distribution at time t:

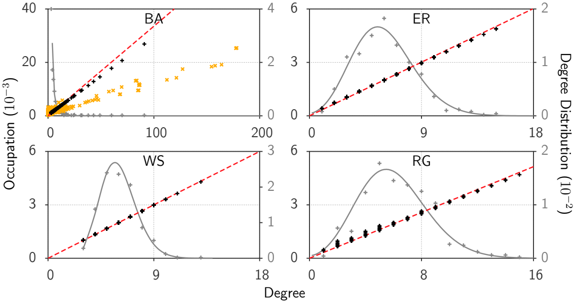
$$(P_Q)_i(t) = |\langle i|\mathrm{e}^{-\mathrm{i}H_Q t}|\Psi_0\rangle|^2$$

The eigenvector corresponding to the zero eigenvalue is:

$$\bar{\phi}_0 = \begin{pmatrix} \sqrt{d_1} \\ \sqrt{d_2} \\ \vdots \\ \sqrt{d_n} \end{pmatrix}$$

Probability Distribution

$(P_Q)_i$ vs. $(P_C)_i$



Quantum Correction

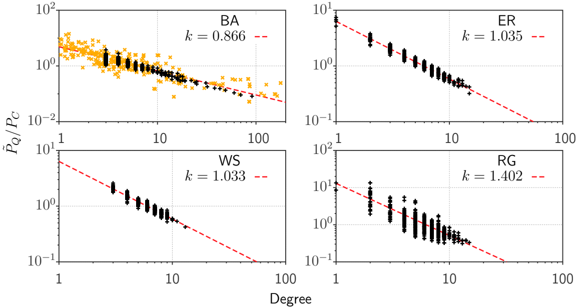
$$(P_Q)_i = \varepsilon(\tilde{P}_Q)_i + (1-\varepsilon)(P_C)_i$$

where:

$$\varepsilon = 1 - \frac{\langle\sqrt{D}\rangle^2}{\langle D\rangle}$$

net.	ε
BA	0.130
ER	0.043
WS	0.016
RG	0.040

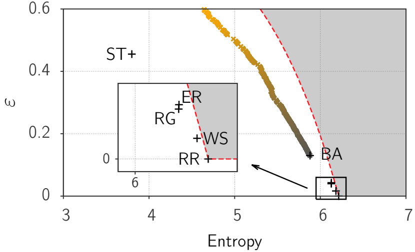
Inhomogeneous degree distribution increase ε



Entropy Bound

The Shannon Entropy of the normalized degree shows how much the degree distribution is homogeneous.

$$\varepsilon \leq 1 - \frac{e^{S_D}}{N}$$



Recap

- Relation of Quantum Walk with Topology (Degree Distribution)
- Separation of Quantum Walk in Classical Part and Quantum Correction through a topological parameter
- Bound to the Quantum correction

Thanks to:

- Jacob Biamonte
- Tomi Johnson
- Piotr Migdal
- Sabre Kais