

Quantum Mechanics and Complex Networks

UCLouvain 2016

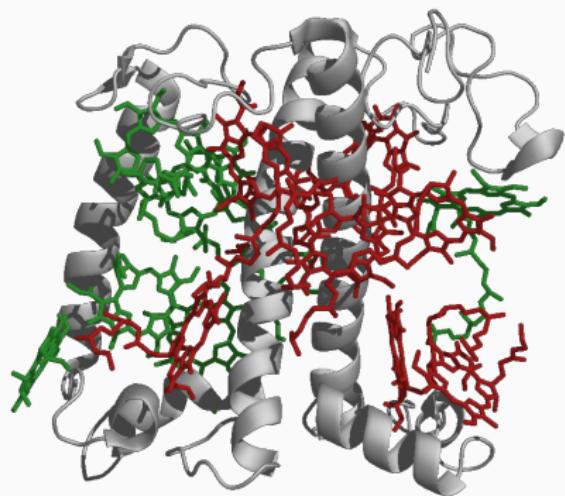
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October 11, 2016



ictteam

Motivation



Spin Logic

Hamiltonians

$$H = \sum_i c_i \sigma_i + \sum_{ij} c_{ij} \sigma_i \sigma_j$$

Pauli Matrices

we consider only
the third

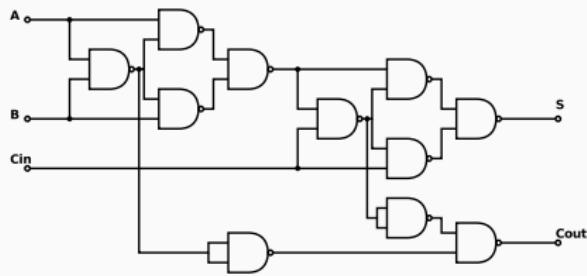
$$\sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Local energy,
depending on spin
configuration
(external fields)

Two body
interaction
defined by the
matrix c_{ij}

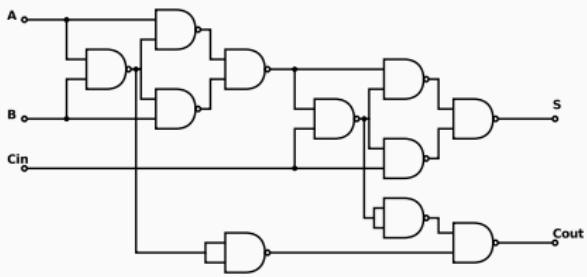
The Problem

Original Problem



The Problem

Original Problem



We map the original problem to finding the ground state of an Hamiltonian H

binary logic gates

$$\begin{array}{ccc} \text{NAND gate symbol} & \Rightarrow & H_{\text{NAND}} \\ \text{NOR gate symbol} & \Rightarrow & H_{\text{NOR}} \end{array}$$

simple Hamiltonian

The Mapping

Truth table for
AND gate:

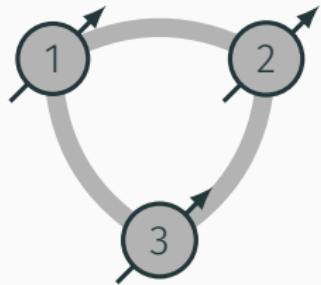
X	Y	$X \wedge Y$
0	0	0
0	1	0
1	0	0
1	1	1

The Mapping

Truth table for
AND gate:

X	Y	$X \wedge Y$
0	0	0
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1	0	0
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$$\begin{aligned} X &\rightarrow (1 - \sigma_1)/2 \\ Y &\rightarrow (1 - \sigma_2)/2 \\ X \wedge Y &\rightarrow (1 - \sigma_3)/2 \end{aligned}$$

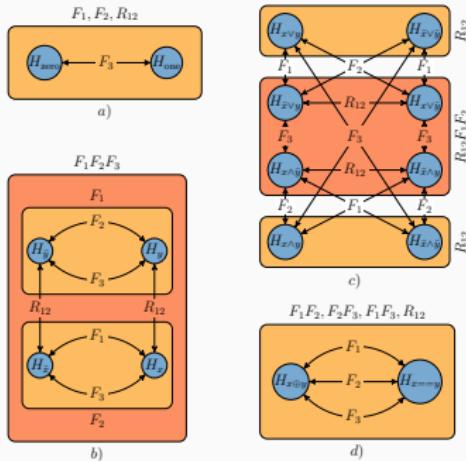


Map to the ground-state of:

$$H(c_i, c_{ij}) = \sum_i c_i \sigma_i + \sum_{i < j}^3 c_{ij} \sigma_i \sigma_j$$

Spin Logic

Symmetries (for optimization)



Hamiltonians:

$$\text{Constants} \quad H_{\text{zero}} = (\mathbb{1} - \sigma_3)$$

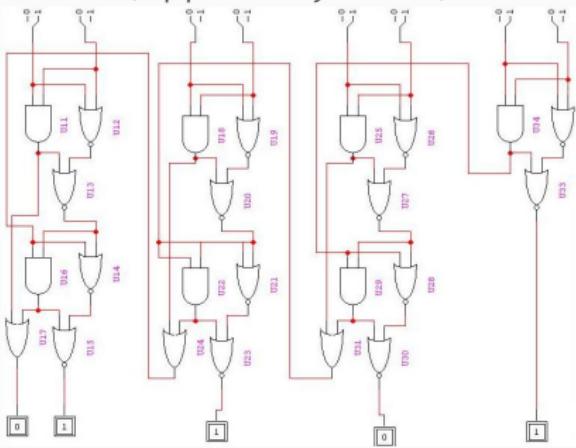
$$Z = X \quad H_X = (\mathbb{1} - \sigma_1\sigma_3)$$

$$\text{AND, OR, ...} \quad H_{\bar{x}\vee\bar{y}} = (c_1\sigma_1 + c_2\sigma_2)(\mathbb{1} + \sigma_3) \\ + (c_1 + c_2)\sigma_3 + c_{12} \sum_{i < j}^3 \sigma_i\sigma_j$$

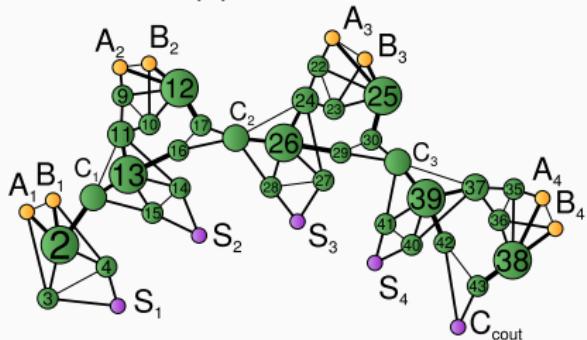
$$\text{XOR EQUIV} \quad H_{x\oplus y} = H_{\bar{x}\wedge\bar{y}}(c_1, c_2, c_4) \\ - \sigma_3 + \sigma_1\sigma_3 + \sigma_2\sigma_3 + \sigma_3\sigma_4$$

Spin Logic

Original Problem
(ripple carry adder)



Mapped Problem



$$H = \sum_i c_i \sigma_i + \sum_{ij} c_{ij} \sigma_i \sigma_j$$

J. D. Whitfield, M. Faccin, and J. D. Biamonte. "Ground-state spin logic". In: *Europhysics Letters* 99.5 (2012), p. 57004. arXiv: 1205.1742 [quant-ph]

Quantumness

Continuous-time Random Walk

	Classical	Quantum
state	$p(t) \in L^1$	$\psi(t) \in L^2$
generator ¹	$H_C = LD^{-1}$	
dynamics	$p(t) = e^{-H_C t} p(0)$	
occup. prob.	$p_i(t)$	
average occup.	π_i^C	π_i^Q

¹ $L = D - A$

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dynamics	$p(t) = e^{-H_C t} p(0)$	$\psi(t) = e^{-iH_Q t} \psi(0)$
occup. prob.	$p_i(t)$	$ \psi_i(t) ^2$
average occup.	π_i^C	π_i^Q

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Steady State

Classical case (master equation):

$$\dot{p}(t) = -H_C p(t)$$

Eigenvector of $H_C = LD^{-1}$ with eigenvalue equal to zero (v_0^C).

$$LD^{-1}v_0^C = 0 \rightarrow \pi_c \propto v_0^C = D\mathbb{1}$$

Steady State: Quantum case

In unitary evolutions there is no steady state: long-time average distribution.

$$(\pi_Q)_i = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt |\psi_i(t)|^2$$

Long time \rightarrow mixture of non-interfering eigen-spaces

$$(\pi_Q)_i = \sum_{\alpha} |(\Pi_{\alpha} \psi(0))_i|^2$$

but $H_Q = D^{-\frac{1}{2}} H_C D^{\frac{1}{2}}$, (similarity transformation) same spectrum
 \rightarrow eigenvectors: $v_{\alpha}^Q \propto D^{\frac{1}{2}} \mathbb{1} = D^{-\frac{1}{2}} v_{\alpha}^C$

Quantumness

Given:

$$(\pi_Q)_i = \sum_{\alpha} |(\Pi_{\alpha}\psi(0))_i|^2$$

Separating subspace relative to the zero eigenvalue:

$$(\pi_Q)_i = (1 - \varepsilon)(v_0^Q)_i^2 + \sum_{\alpha > 0} |(\Pi_{\alpha}\psi(0))_i|^2$$

Quantumness:

$$\pi_Q = (1 - \varepsilon)\pi_C + \varepsilon\tilde{\pi}_Q$$

Graph Quantumness

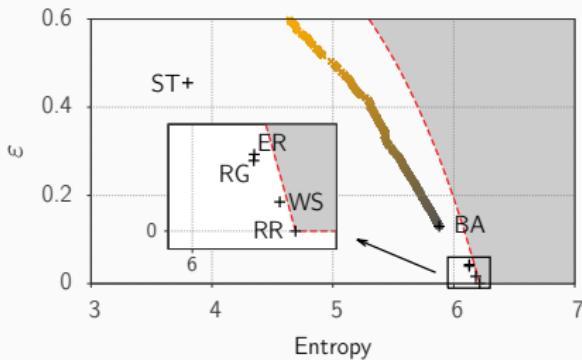
- At zero temperature, quantum is classic
- At higher temperature quantum effects on the long time average
- At fixed temp., higher quantum effects on heterogenous topologies

For a *flat* initial state:

$$\varepsilon = 1 - \langle \sqrt{d} \rangle / \langle d \rangle$$

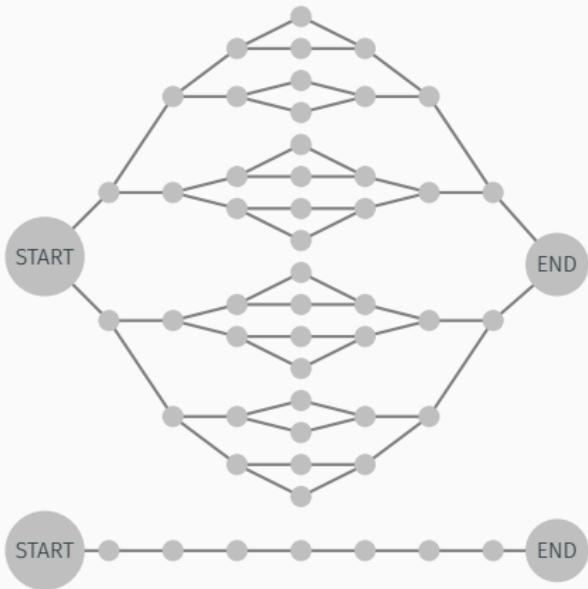
upperbound:

$$\varepsilon \leq 1 - N^{-1} e^{H_1(\{d_i / \sum_i d_i\})}$$



Chiral Quantum Walks

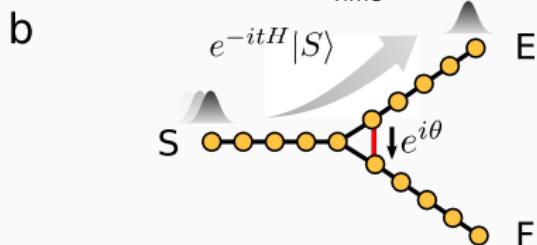
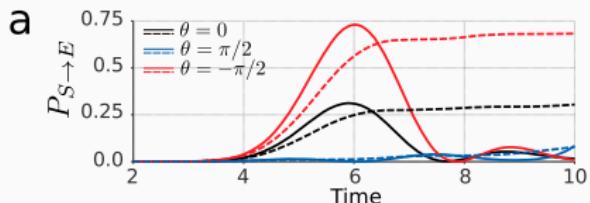
Quantum Walks



- Quantum walks get exponential speedup in the G4 graph.
- The quantum walker (particle) just sees a chain.
- $H = A$
- usually symmetric and (hence real) Hamiltonians

Biased Quantum Walks

Can we beat *normal* quantum walks?



Direction of the dynamics can be controlled by external fields

Beat Quantum Walks

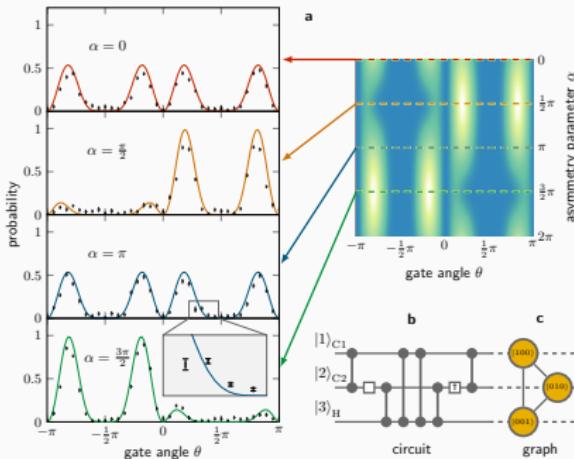
Time symmetry breaking
on a real experiment
(NMR).

Topology effects:

Trees No effect on probability

Even Loops Effect only on probability

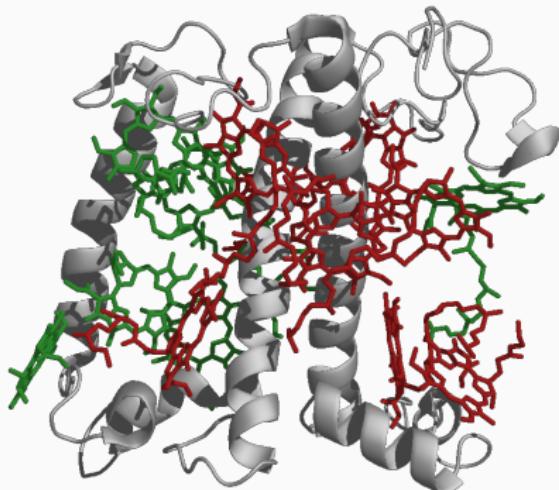
Odd Loops Effects probability and time-symmetry



Z. Zimboras et al. "Quantum Transport Enhancement by Time-Reversal Symmetry Breaking". In: *Sci. Rep.* 3 (2013). Ed. by NPG, p. 2361. arXiv: 1208.4049 [quant-ph]

D. Lu et al. "Chiral quantum walks". In: *Phys. Rev. A* 93 (4 2016), p. 042302

Quantum Communities



Complex Structure

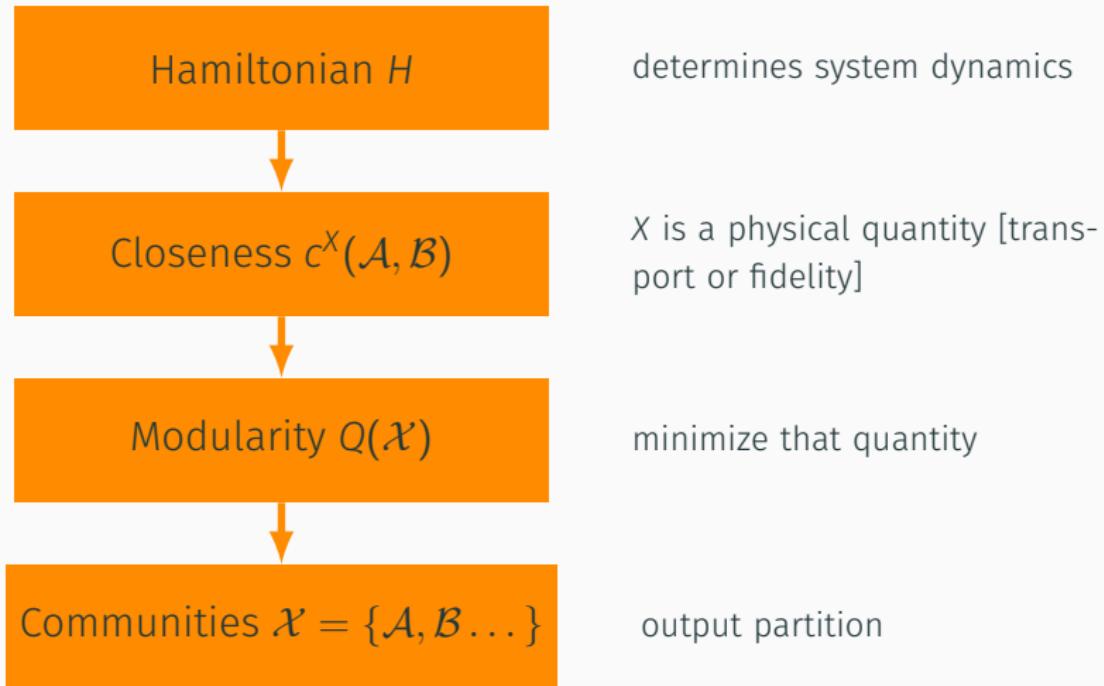
Cases:

- Energy transport
- Quantum communication

Goals of the approach:

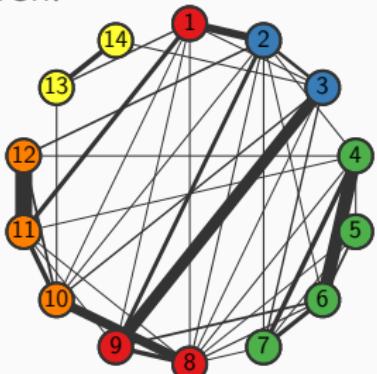
- Simulation
- Design

Our Approach

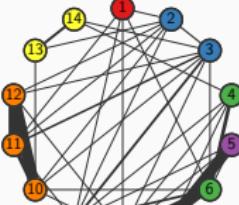


Light harvesting systems

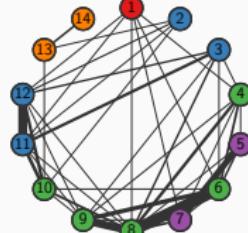
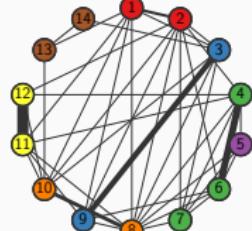
LHCII:



Transport



Fidelity

Only interaction
strength

(classical)

M. Faccin, P. Migdał, T. H. Johnson, V. Bergholm, and J. D. Biamonte. "Community Detection in Quantum Complex Networks". In: *Phys. Rev. X* 4 (4 2014), p. 041012. arXiv: 1310.6638 [quant-ph]

Thanks

The Team



Tomi Johnson



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