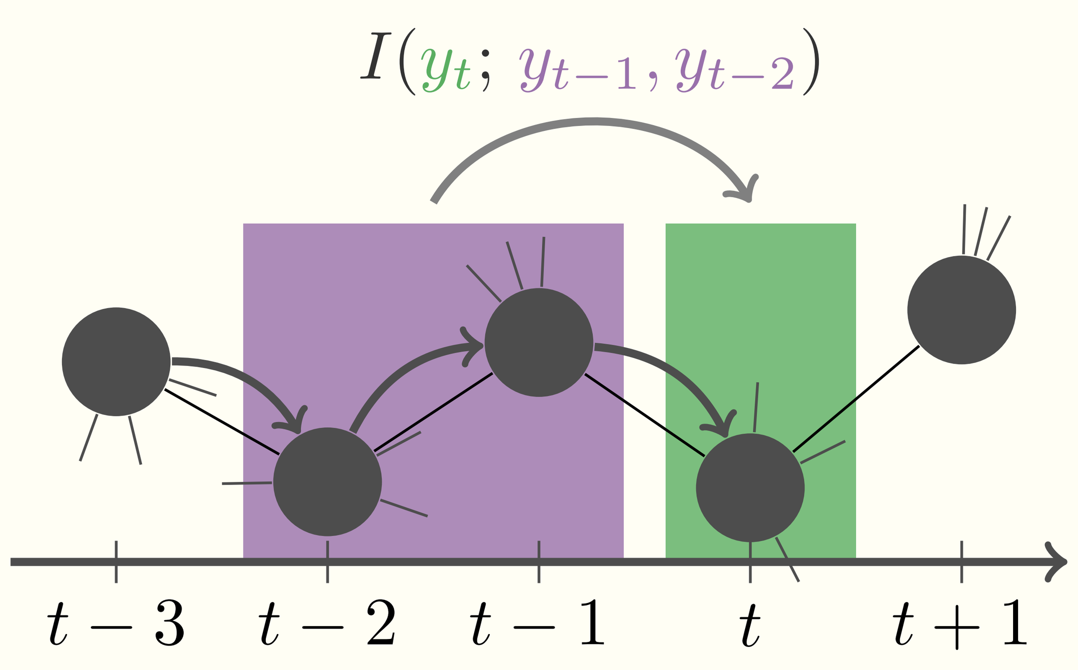
Non-Markovian dynamics

In a complex dynamical system, large part of **the apparent complexity may be due to its non-Markovian dynamics**.

To simplify a complex dynamical system, we need to take into account both **its structure and its dynamics**.
E.g. *web browsing*, *flight network* or *e-mail exchange* may be considered systems with *complex non-Markovian* dynamics.

- Information flows [mainly] from past (history) to present.
- Information flowing from ahead of history is negligible.



To produce a simpler model for a dynamical system we want to aggregate states. *How can this be done?*

Maximize Auto-Information

$$\mathcal{I}_b = I(y_t; y_{t-b})$$

Maximize the predicatiblity (information flow), retain the Markov order.

TL,DR

To find a simplified model of a complex dynamical system, we need to **take into account both its structure and its dynamics** which may be non-Markovian.

Aggregating the states of a system according to **auto-information (AISA)**, i.e., partitioning the network, **produces a simplified model** of the original system.

A simple Markov chain with the aggregated states, will also produce **dynamics that better approximates the original systems' dynamics**.

We found similar results in a number of real networks.

State Aggregation in non-Markovian systems
by Auto-Information maximization

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Does state aggregation lead to a good dynamical model?

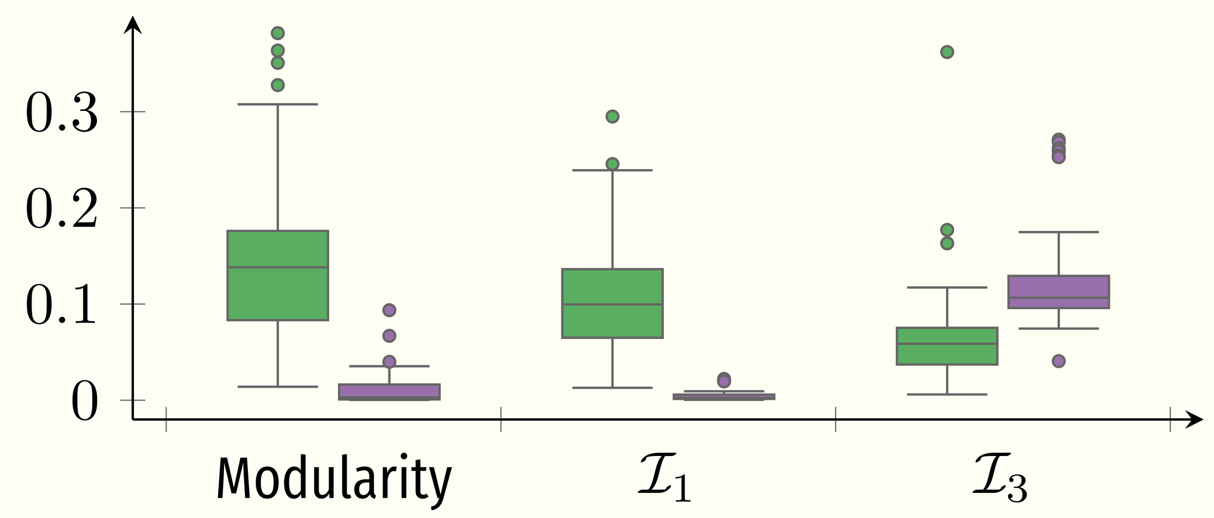
Simulation of the original dynamics through Markov dynamics (random walk) on the partition network.

Average error of the simulated dynamics compared to the original green:

- random walk on the *modularity* partition diverges from the real dynamics (high errors);
- random walk on the $\mathcal{I}_1$ partition (Markovianity assumption) diverges from the real dynamics;
- random walk on the $\mathcal{I}_3$ partition is the **closest** to the real dynamics;

Notice the opposite behaviour when comparing to a random walk on the original network purple.

Error of the simulated dynamics:



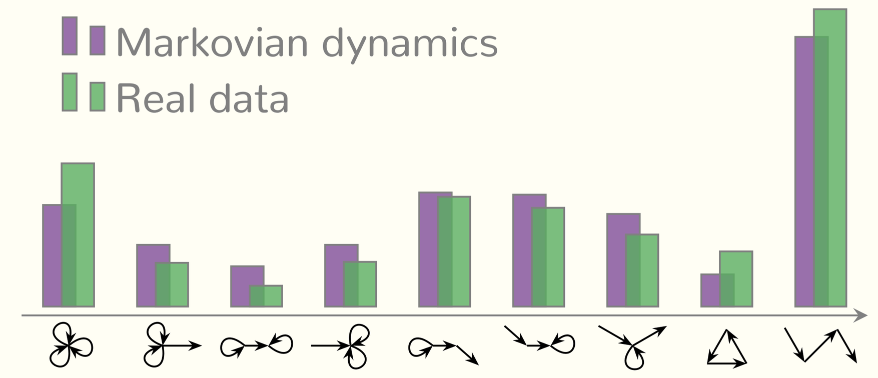
The simplified model produced by AISA ($\mathcal{I}_3$ maximization) with a simple first order Markov chain best reproduces the original higher order Markovian dynamics.

State aggregation on a real network

Browsing behaviour of web portal users (TV Broadcasting).

Dynamical patterns contained in the real dynamics or in the first order Markovian approximation (a random walk).
The random walk model **underestimates** the importance of some dynamical patterns.

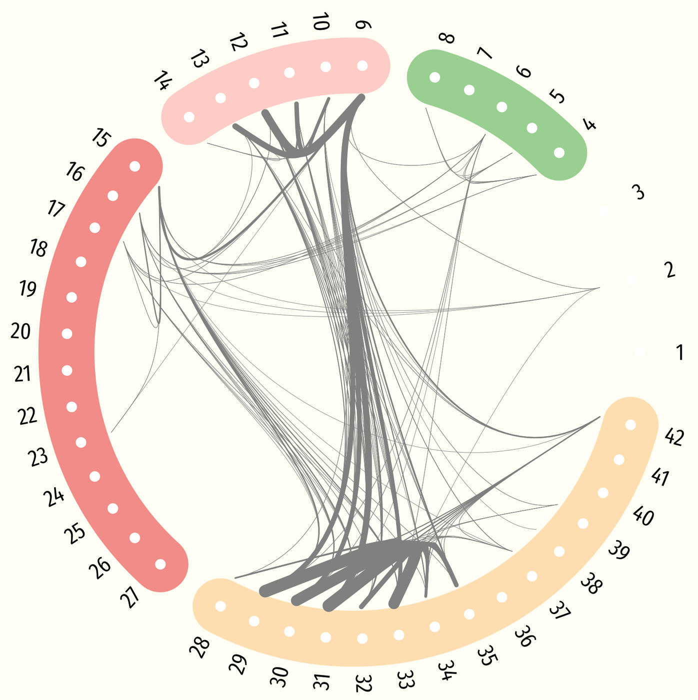
non-Markovianity of the dynamics



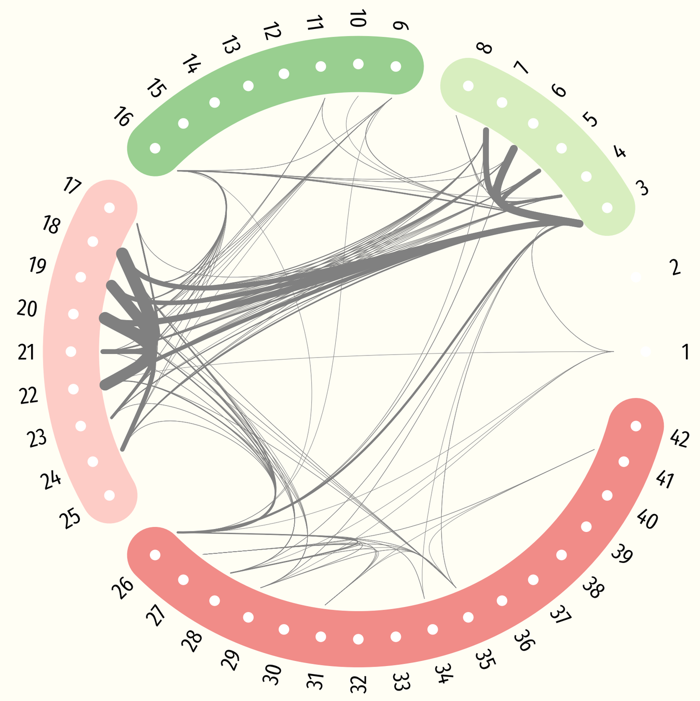
Auto-Information State Aggregation (AISA)

The maximization of the Auto-Information ($\mathcal{I}_3$) or of the modularity bring two similar partitions, however random walks on the partition networks diverge (see box on the left):

AISA ($\mathcal{I}_3$ maximization):



Modularity maximization:



Acknowledgements: MF funded by Innoviris (D1.31402.007-F)