Ground State Spin Logic

Mauro Faccin
Jacob Biamonte, James Whitfield

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Quantum Science Lab
Institute for Scientific Interchange

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Department of Mathematics
Penn State University

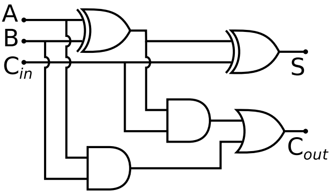
- Simulation of Boolean functions embedded into Hamiltonian ground states;
- Two-spin interaction Hamiltonians can be used to simulate all logical gates;
- Symmetry arguments help to adapt the simulation algorithms to constraints.

Spin Logic

Simulation of a logical circuit through a system of spins.

- All the solutions of the logical circuit will be embedded into the ground state of their corresponding Hamiltonian;
- The ground state subspace only characterizes solutions of the logical circuit.

Circuits / Boolean Functions:



Spin system Hamiltonian:

$$H = \sum_i c_i \sigma_i + \sum_{ij} c_{ij} \sigma_i \sigma_j + \sum_{ijk} c_{ijk} \sigma_i \sigma_j \sigma_k + \dots$$

Spin Logic

Consider the $2^4 = 16$ truth tables of logical gates with two inputs, one output.

$$f: \quad B^2 \to B$$
$$f(x, y) = z$$

Generic Truth Table:

x	y	z
0	0	b_1
0	1	b_2
1	0	b_3
1	1	b_4

where $b_i \in \{0, 1\}$

Symmetry generators

Action on the gates:

- bit flip (F_1, F_2, F_3)
- input swap (R_{12})

$\text{AND} = x \wedge y$

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

Symmetry generators

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- bit flip (F_1, F_2, F_3)
- input swap (R_{12})

$\text{AND} = x \wedge y$

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

$\xrightarrow{F_1}$

x	y	z
1	0	0
1	1	0
0	0	0
0	1	1

Symmetry generators

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- bit flip (F_1, F_2, F_3)
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$\text{AND} = x \wedge y$

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$\xrightarrow{F_1}$

x	y	z
1	0	0
1	1	0
0	0	0
0	1	1

$\xrightarrow{\text{Reorder}}$

$\bar{x} \wedge y$

x	y	z
0	0	0
0	1	1
1	0	0
1	1	0

Symmetry generators

Action on the gates:

- bit flip (F_1, F_2, F_3)
- input swap (R_{12})

AND $= x \wedge y$

x	y	z
0	0	0
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$\xrightarrow{F_1}$

x	y	z
1	0	0
1	1	0
0	0	0
0	1	1

$\xrightarrow{\text{Reorder}}$

$\bar{x} \wedge y$

x	y	z
0	0	0
0	1	1
1	0	0
1	1	0

Spin swaps affecting the third spin, i.e. R_{13} and R_{23}, do not preserve the truth-tables space.

Symmetries

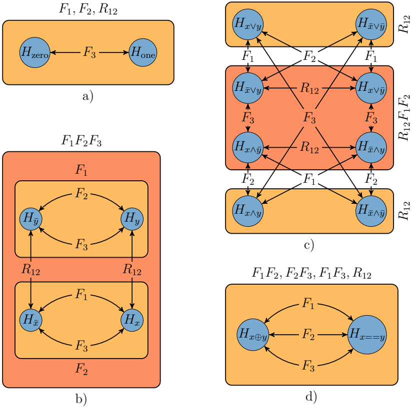
The ground states corresponding to each gate must carry its symmetries.

Actions:

- bit flip (F_i)
- input swap (R_{12})

Generate Four Orbits

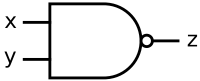
Remark: high energy states do not need to present the same symmetries



Example: The NAND gate

$$\mathrm{NAND}(x,y) = \overline{x \wedge y} = \bar{x} \vee \bar{y}$$

x	y	$z = \overline{(x \wedge y)}$
0	0	$b_1 = 1$
0	1	$b_2 = 1$
1	0	$b_3 = 1$
1	1	$b_4 = 0$



The ground state of the target Hamiltonian have to span the corresponding states:

$$L(H_{\mathrm{NAND}}) = \mathrm{span}\{|001\rangle, |011\rangle, |101\rangle, |110\rangle\}$$

Orbits: Constants

x	y	z=0
0	0	0
0	1	0
1	0	0
1	1	0

$$H_{\text{zero}}(\sigma_3) = -c\sigma_3$$

with $c > 0$

Orbits: $x, y, \bar{x}, \bar{y}$

x	y	z=x
0	0	0
0	1	0
1	0	1
1	1	1

$$H_x(\sigma_1, \sigma_3) = -c\sigma_1\sigma_3$$

with $c > 0$

Orbits: NAND,AND...

x	y	z=NAND(x,y)
0	0	1
0	1	1
1	0	1
1	1	0

$$H_{\mathrm{NAND}}(\sigma_1, \sigma_2, \sigma_3) = (c_1\sigma_1 + c_2\sigma_2)(1 + \sigma_3) + (c_1 + c_2)\sigma_3 + c_{12} \sum_{i<j} \sigma_i\sigma_j$$

with $c_1, c_2, c_{12} > 0$

Orbits: EQUIV,XOR

x	y	z=XOR(x,y)
0	0	0
0	1	1
1	0	1
1	1	0

$$H_{\mathrm{XOR}}(\sigma_1, \sigma_2, \sigma_3) = -c\sigma_1\sigma_2\sigma_3$$

with $c > 0$

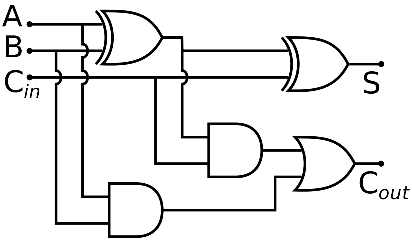
Orbits: EQUIV,XOR

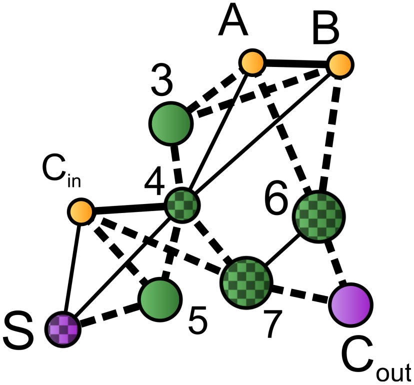
x	y	z=XOR(x,y)
0	0	0
0	1	1
1	0	1
1	1	0

$$H_{\mathrm{XOR}}(\sigma_1, \sigma_2, \sigma_3) = H_{\mathrm{NOR}}(\sigma_1, \sigma_2, \sigma_4) - \sigma_3 + (\sigma_1 + \sigma_2)\sigma_3 + 2\sigma_3\sigma_4$$

with $c_1, c_2, c_{12} > 1/2$

Example One: Full Adder

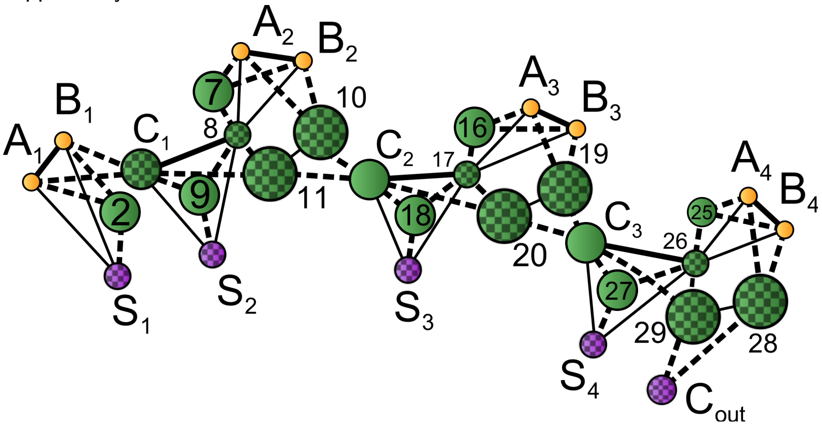




10 spins, 5x3=15 parameters.

ExampleTwo: You Can Go Big

Ripple carry adder with 4 bits.



32 spins, 51 free parameters.

- Full description of two-input one-output gates
- Description of the Symmetry group acting on each gate provide flexibility in the choice of the parameters
- Simulation of three-body interaction via two-body Hamiltonians

Thanks To:

James Whitfield

Vienna Center For Quantum Science and Technology,
Boltzmanngasse 5 1090 Vienna, Austria

Jacob Biamonte

Institute for Scientific Interchange,
Via Alassio 11/c, 10126 Torino, Italy