Complex systems

a dynamical point of view

Mauro Faccin

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AutoInformation state aggregation

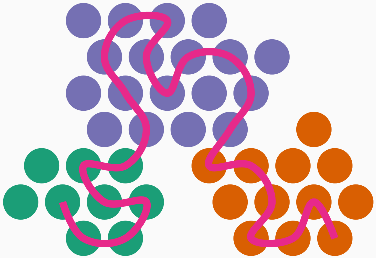
Projected Markov Chain

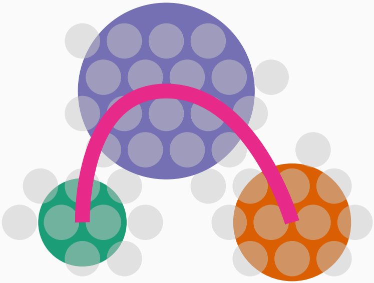
Markov Chain

$$\ldots, \mathbf{x}_{\text{past}}, \mathbf{x}_{\text{now}}, \mathbf{x}_{\text{future}}, \ldots$$

Projection

$$\ldots, \mathbf{y}_{\text{past}}, \mathbf{y}_{\text{now}}, \mathbf{y}_{\text{future}}, \ldots$$





Non-linear correlations

AutoInformation

$$\mathbf{I}(\mathbf{y_t};\mathbf{y_{t-\tau}})$$

Non-linear correlation between successive time-steps

$$\mathbf{I}(\mathbf{y_t};\mathbf{y_{t-\tau}}) = \underbrace{\mathbf{I}(\mathbf{y_t};\mathbf{y_{t-\tau}},\dots)}_{^{1}\text{Predictability}} - \underbrace{\mathbf{I}(\mathbf{y_t};\mathbf{y_{t-2\tau}},\dots|\mathbf{y_{t-\tau}})}_{^{2}\text{non-Markovianity}}$$

where τ represents a time-scale parameter.

We ask to:

1. maximize predictability of the projected dynamics;
2. minimize non-Markovianity (memories arising from the projection).

Faccin et al, Journal of Complex Networks, 2018

Faccin et al, PRL, 2021

Links to the literature

Modularity

χ_c characteristic function of class c

$$Q = \sum_c \mathrm{Cov}\left(\chi_c(t), \chi_c(t+1)\right)$$

Auto-covariance of the dynamics on the partition space.

Linear correlation between consecutive time-steps.

Shen et al. (2010) PRE, 82, 016114

DC-SBM

$$\mathcal{S} \propto \frac{1}{2} \sum_{cd} e_{cd} \log \frac{e_{cd}}{e_c e_d}$$

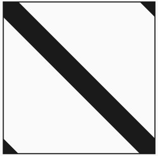
Fitting a generative model (e.g. DC-SBM) to the data through log-likelihood maximization can be seen as maximizing the AutoInformation for paths of lenght $\tau = 1$ (e.g. links).

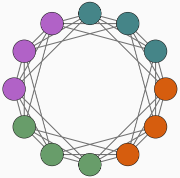
Karrer and Newman (2011), PRE 83, 016107.

Example: A cycle

A regular ring lattice with **N** nodes, each connected with **k** neighbours.

How many classes?

Adj: 



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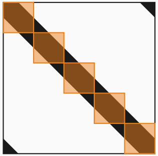

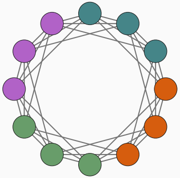

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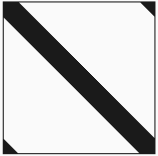
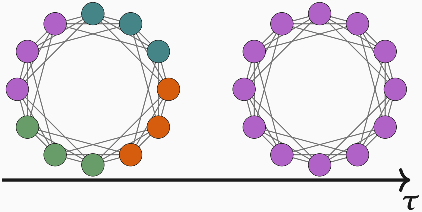
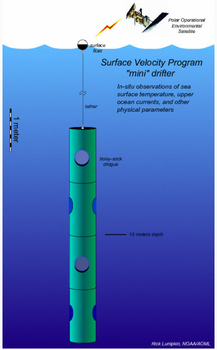


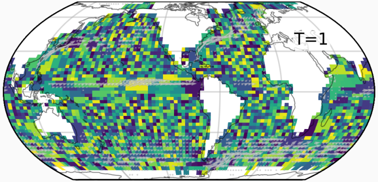


Example: Ocean buoys



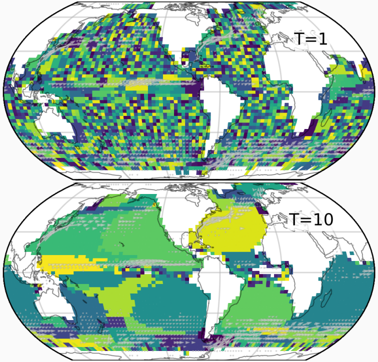
Global Drifter Program

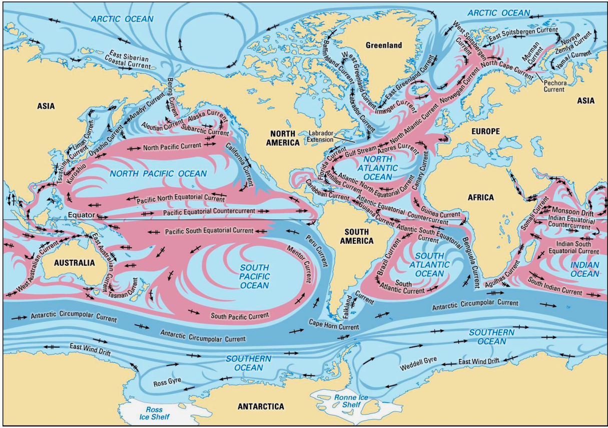
Example: Ocean buoys



Each time step lasts 16 days.

Example: Ocean buoys





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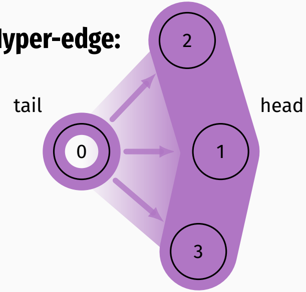
Dynamics on hypergraphs

Hyper-edge:



node	role
0	tail
1, 2, 3	head

Hypergraph: $\{\mathbf{N}, \mathbf{E}\}$: nodes and hyperedges

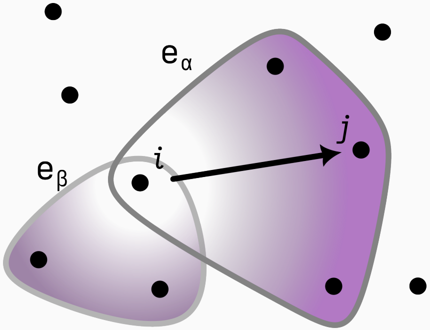
Nodes: same as before

Hyperedges: $\mathbf{e}_\alpha = \{\text{tail}, \text{head}\} \in \mathbf{E}$

Random walker on a hypergraph

The walker

- sits on a node **i**
- chooses a hyperedge $\mathbf{e}_{\boldsymbol{\alpha}}$ incident on **i** in its tail (user **i** tweeted $\boldsymbol{\alpha}$) with probability dependant on the hyperedge size (with parameter $\boldsymbol{\tau}$);
- chooses an exit node **j** from the head of $\mathbf{e}_{\boldsymbol{\alpha}}$ ($\boldsymbol{\alpha}$ get retweeted by user **j**) with flat probability.

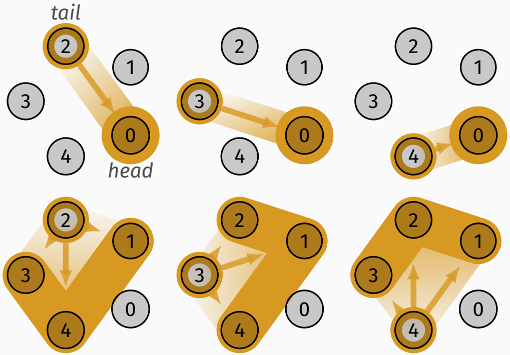


Yes but why?

Measure the dynamics through its transition matrix instead of extending to the hypergraph framework with the corresponding complexity overhead.

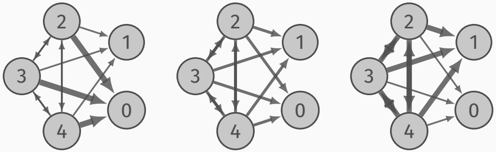
↓ Ranking nodes

Hyper-Edges



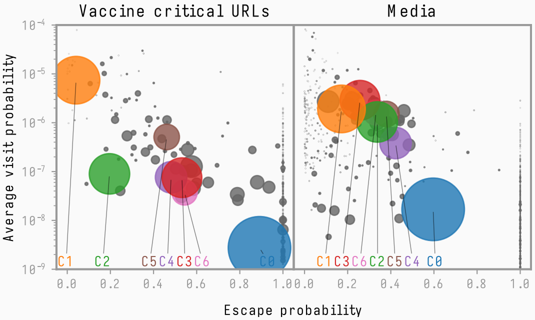
$\tau = -1$		$\tau = 0$		$\tau = 1$	
ranking	prob	ranking	prob	ranking	prob
0	0.325	1	0.224	1	0.251
1	0.186	0	0.224	2	0.2
2	0.163	2	0.184	3	0.2
3	0.163	3	0.184	4	0.2
4	0.163	4	0.184	0	0.149

Effective graphs



Faccin, PRE, 2022

Sharing of URLs to blogs and sites critical to the use and development of vaccines.



Visiting probability probability of being visited by a random walker

Escape probability probability of reaching other communities

Communities detected by **Stability**

Comm.	**Interpretation**
C_0	media aggregators or web influencers.
C_1	Far right groups.
C_2	health institutions and MDs
C_3	French news media.
C_4	international news media.
C_5	Far left and trade unions.
C_6	government representatives.
C_7	Canada

C_1 and C_5 are the main actors in spreading vaccine-critical content.

Delvenne et al, PNAS, 2010

Compartmental models

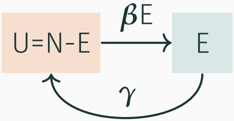
SIS Compartmental model

Engagement

Engaged users share a URL from the set within a time window $\tau = 3$ days.

$$\mathbf{dE_t} = \alpha_t \frac{\mathbf{E_t}(\mathbf{N_t} - \mathbf{E_t})}{\mathbf{N_t}} - \beta_t \mathbf{E_t}$$

$$\mathbf{R_t} = \frac{\alpha_t}{\beta_t}$$

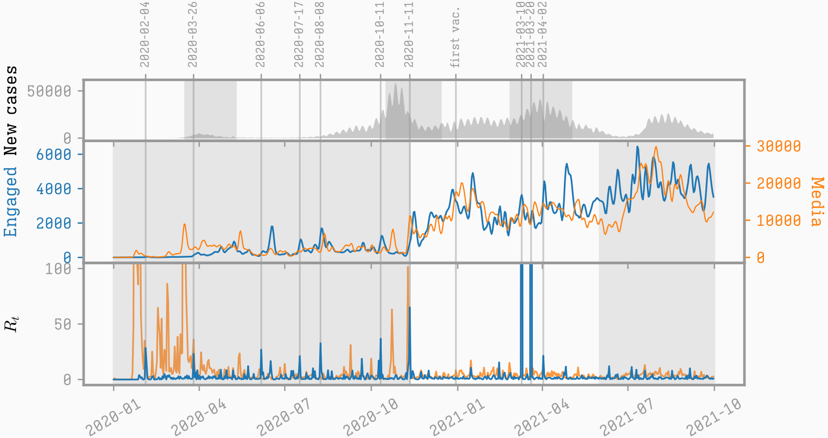


α_t engagement rate

β_t unengagement rate

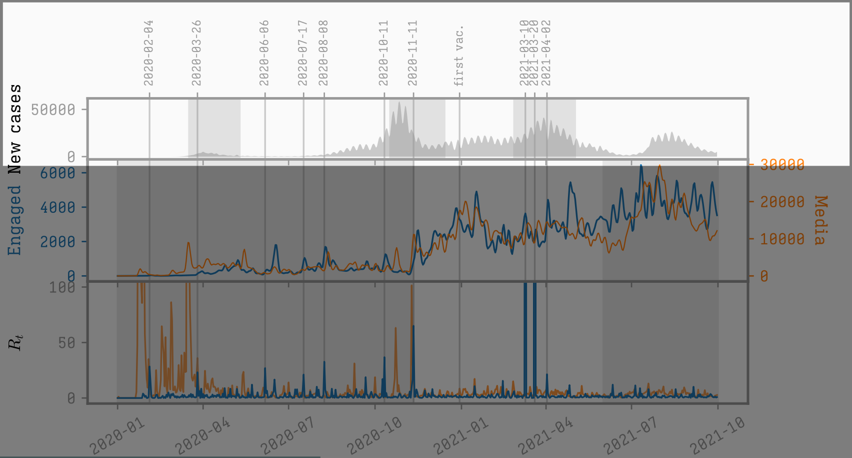
$\mathbf{R_t}$ reproduction number

Engagement evolution



Faccin et al., PLOS ONE, 2022

Engagement evolution



Faccin et al., PLOS ONE, 2022

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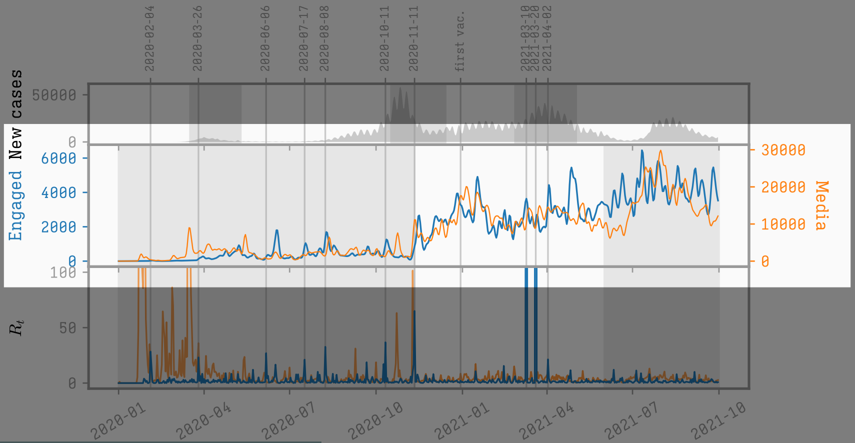

Faccin et al., PLOS ONE, 2022

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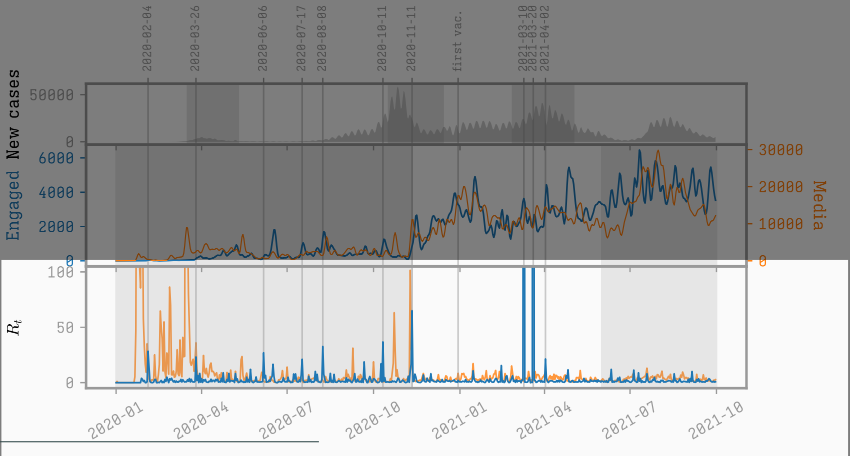

Faccin et al., PLOS ONE, 2022

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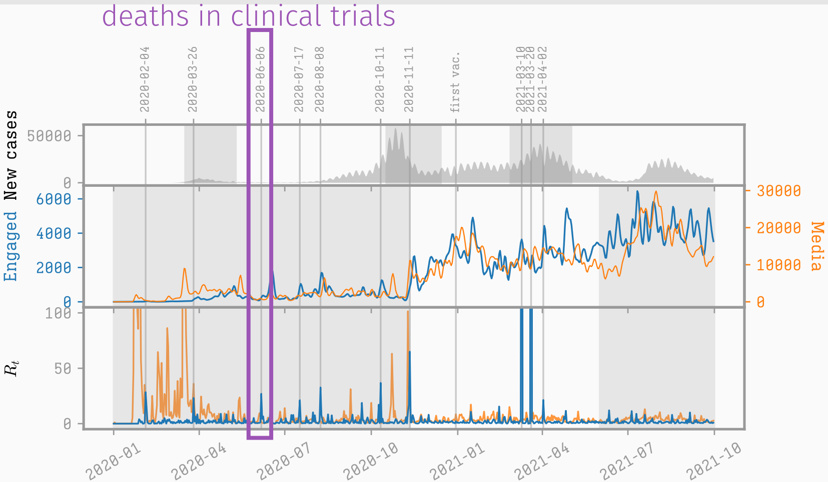

Faccin et al., PLOS ONE, 2022

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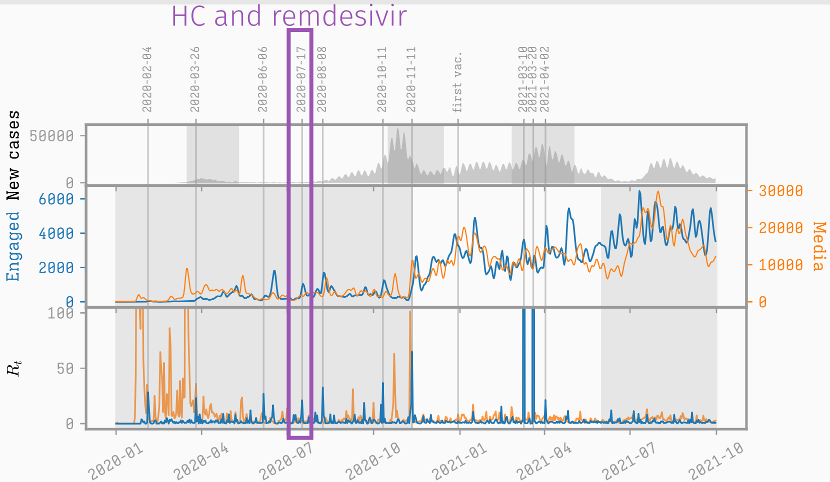

Faccin et al., PLOS ONE, 2022

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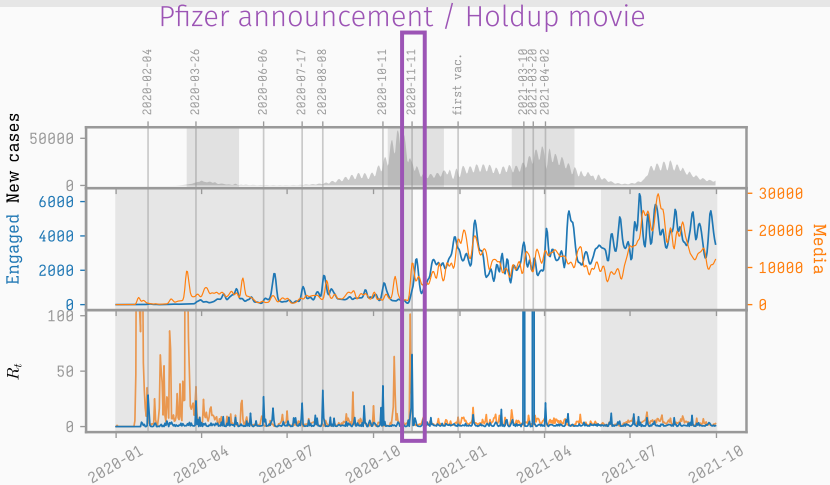

Faccin et al., PLOS ONE, 2022

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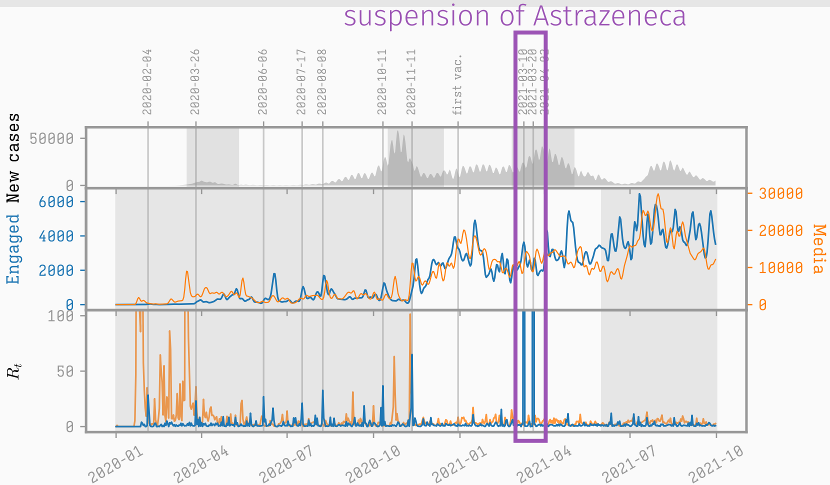


Faccin et al., PLOS ONE, 2022

Tuberculosis under-detection

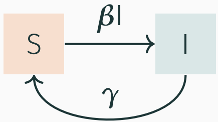
- 4 million of undetected/untreated TB cases, yearly [WHO]
- Hard to reach communities with high levels of TB incidence

Efficient approach to ACF

Data driven estimation of TB incidence rates to focus health-care interventions such as Active-Case Findings (ACF)

Active-Case Finding: systematic screening of the population for active TB cases.

Use of compartmental models (SIS) to disaggregate the reported cases.



Assumption

Endemic disease with slowly evolving **well mixed population**

$$\begin{cases} \frac{dS}{dt} &= \gamma I - \beta \frac{IS}{S+I} \\ \frac{dI}{dt} &= \beta \frac{IS}{S+I} - \gamma I \,. \end{cases}$$

$\longrightarrow$ fit the parameters to:

- the number of cases reported by the **local health system** subunits;
- the population density as estimated by [Worldpop]

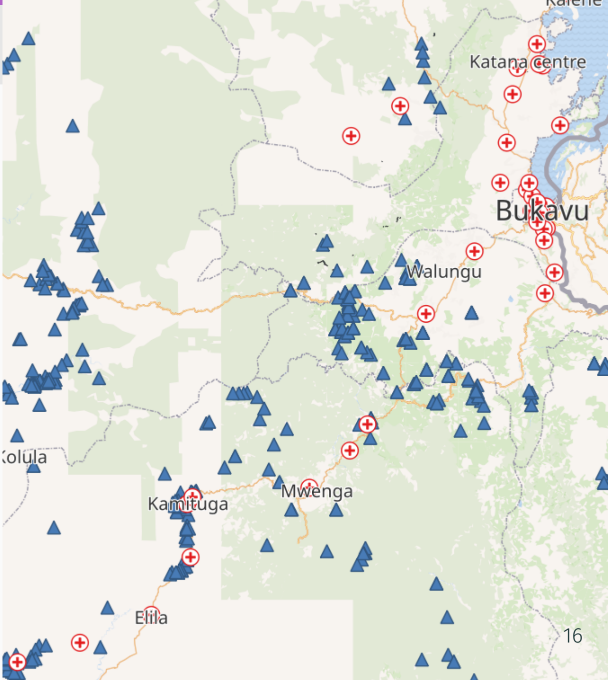
Model refinement

We include additional data to refine the estimation:

- Mine locations;
- Health facilities,

from of openly available sources:

- OpenStreetMap
- IPIS Research Project (mines)
- Healthsites

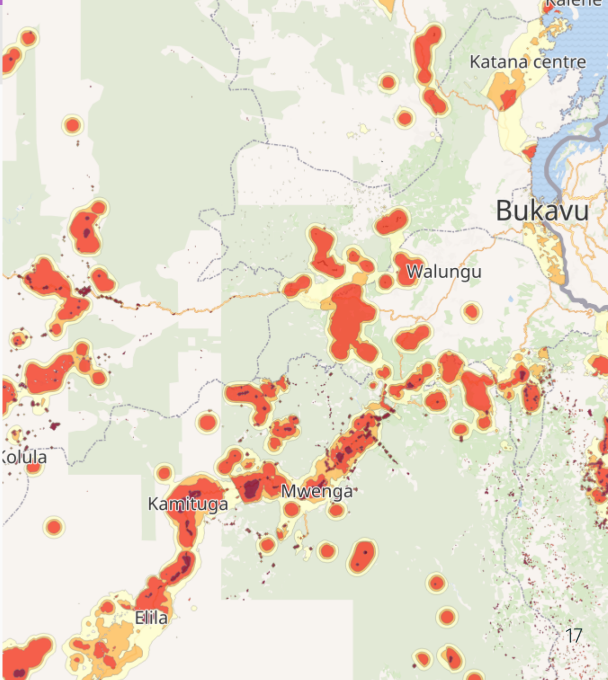


Incidence rate estimation

The estimated incidence rates highlight the location of population pockets with high risk of TB.

> Leads to efficient ACF interventions.

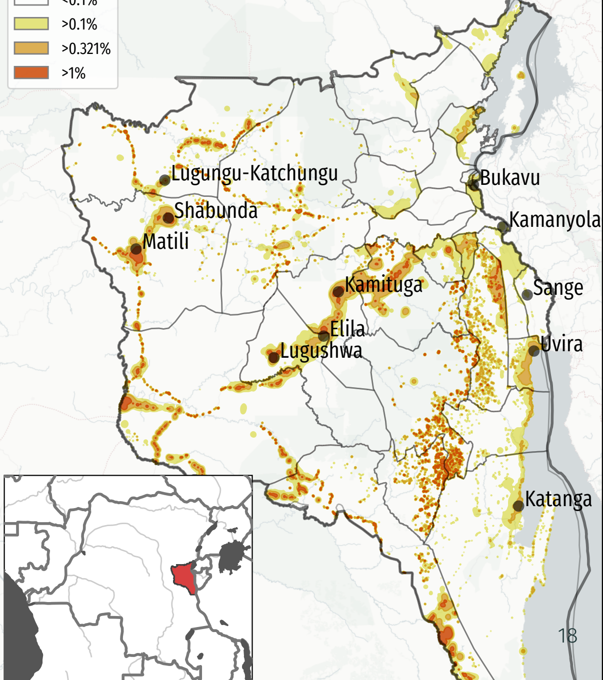
color	incidence rate
(light yellow)	>0.1%
(orange)	>0.32%
(red)	>0.5%
(dark red)	>1%



Multicentric clinical trial

We performed a ***multicentric clinical trial*** addressing 11 locations with heterogeneous estimated incidence rates.

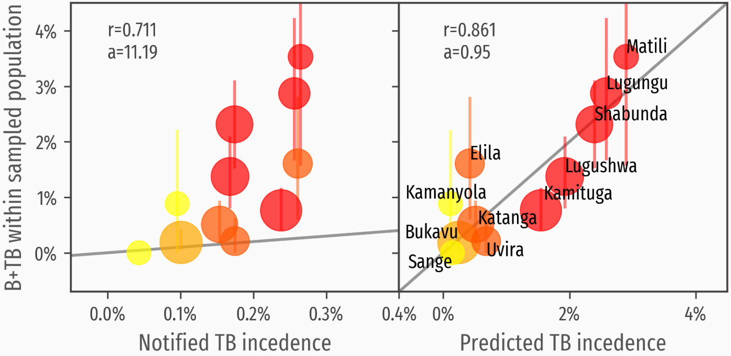
Faccin et al, Sci Rep, 2022



Results

screenings	13.841
lab tests	1153
positive cases	112

> **80%** of positive cases originated from locations at high risk
(estimated incidence rate higher than 1%).



🏃 Finally...

Questions?

Joint work with:


JC Delvenne

UCLouvain


M Schaub

RWTH AACHEN UNIVERSITY


F Gargiulo

SORBONNE UNIVERSITÉ


J Ward

Université Paris Cité


E. André

UZ LEUVEN

https://maurofaccin.github.io
mauro.fccn@gmail.com

Code at:

https://maurofaccin.github.io/aisa
https://maurofaccin.github.io/cartotb