# The variability of network structures inferred from time series data

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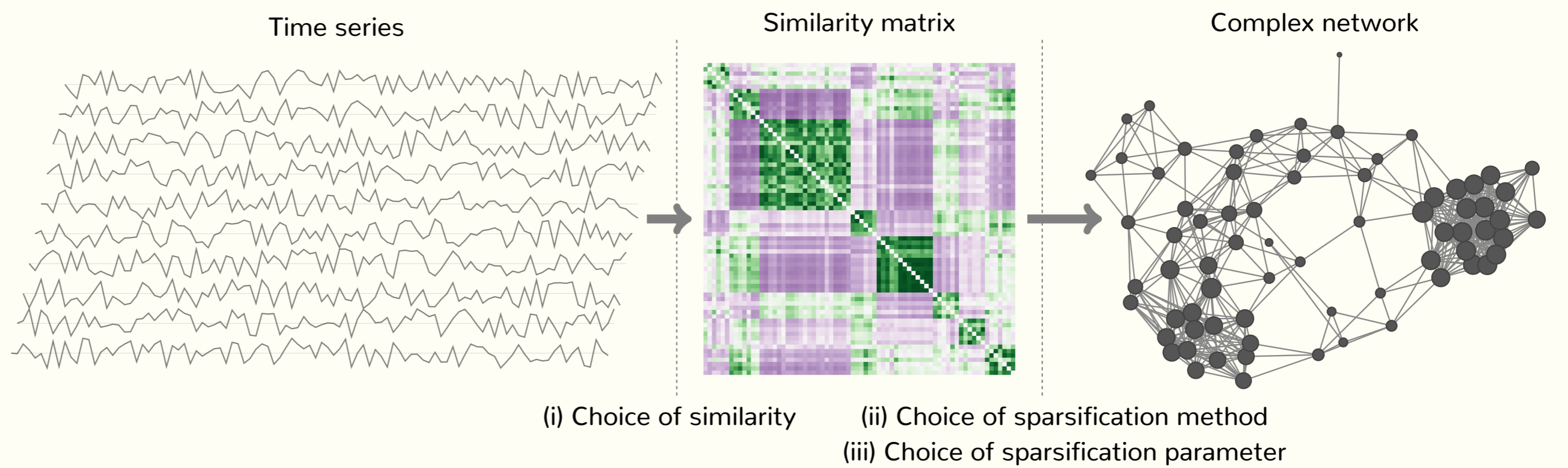
## Network reconstruction

Many choices influence network reconstruction from temporal data.
E.g.:

**Similarity** The time series similarity (e.g. **Pearson's correlation**)

**Sparsification** The method used to select edges from a full matrix (e.g. **thresholding**, **K-nearest neighbours**, **graphical Lasso**)

Here we focus on Pearson's correlation.



Time series → Similarity matrix → Complex network

(i) Choice of similarity
(ii) Choice of sparsification method
(iii) Choice of sparsification parameter

## TL,DR

We collected a number of time–series from different fields:

**fMRI data** (neuroscience);

**S&P100 price fluctuations** (finance);

**daily temperature of US cities** (meteorology);

**Tuberculosis incidence reports** (epidemiology);

**Synthetic data**

We show that network **statistics highly depend on the choice of sparsification method and the value of its parameter.**

- Linear correlation methods do not catch the underlying structure.
- Network reconstruction needs careful consideration and clear assumptions.
- Further work is necessary to link analysis of temporal datasets to complex network techniques.

## Fluctuations on real networks

Density range depends on the sparsification method.
Network stats depends on the sparsification parameter.



Density and various statistics.
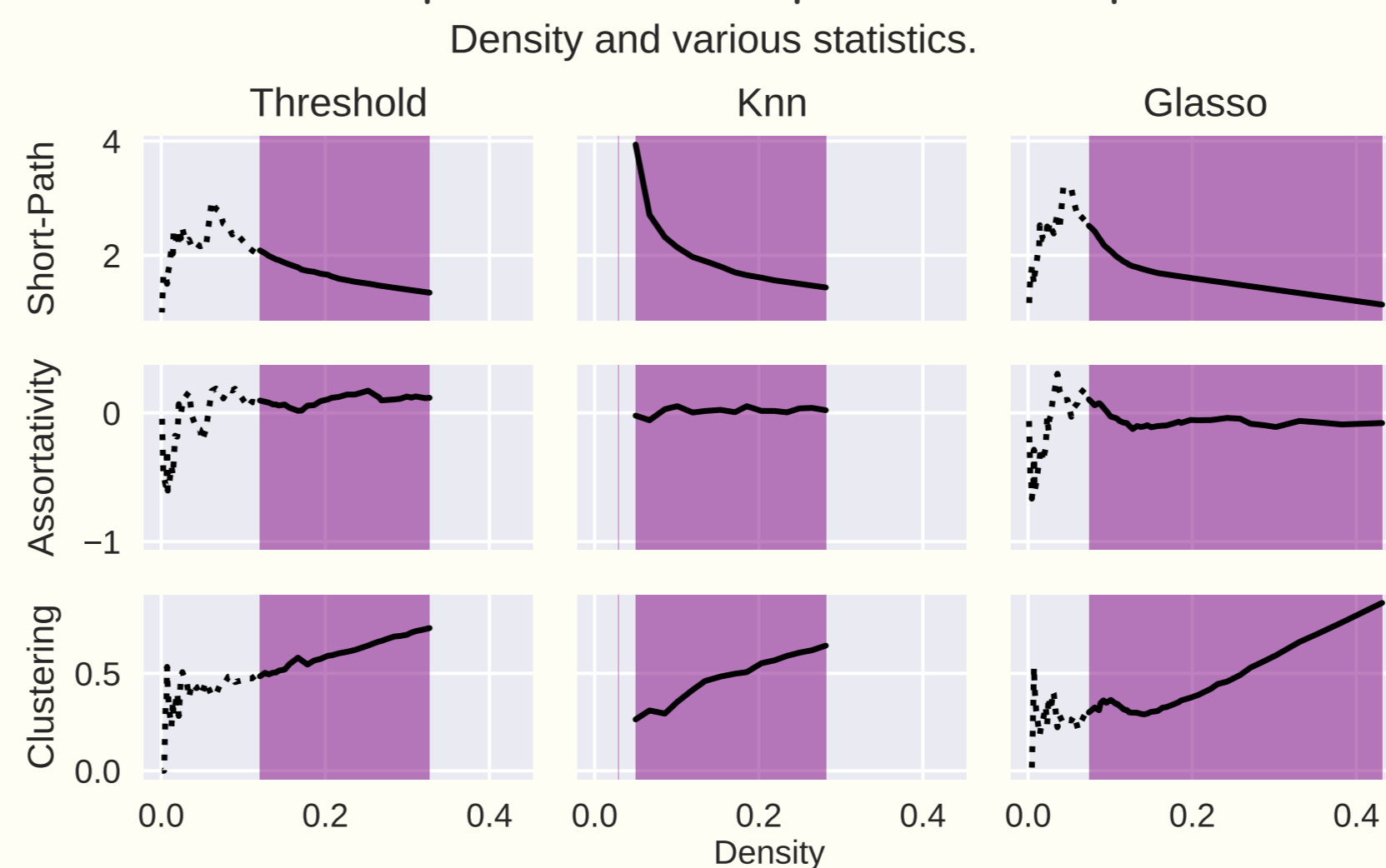
Tuberculosis reports on health zones of DRC from 2010 to 2017.
In purple the density range for which each method returns a connected graph.

### Selection of parameters by **robustness**
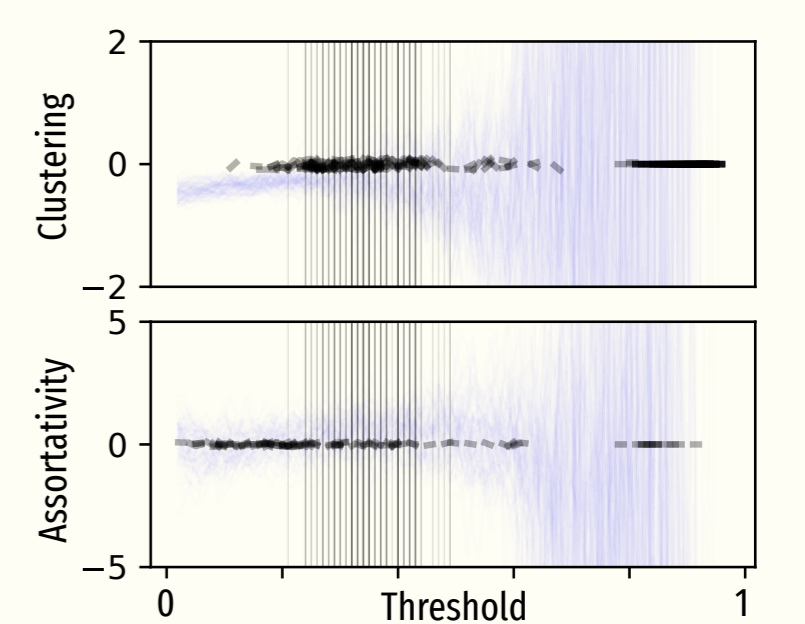


Discrete derivatives (blue lines).

Plateaux (black dots) of different measures appear in different parameter regions (here with thresholding).

Vertical lines denote the parameter at which each network become disconnected.
fMRI data of resting subjects.

## Synthetic analysis

A simple dynamical system (diffusion + noise) evolves on a known network topology. The reconstructed network is compared to the original one.



Complex network → Dynamics $\dot{x} = \mathbf{L}x + \delta$ → Time series
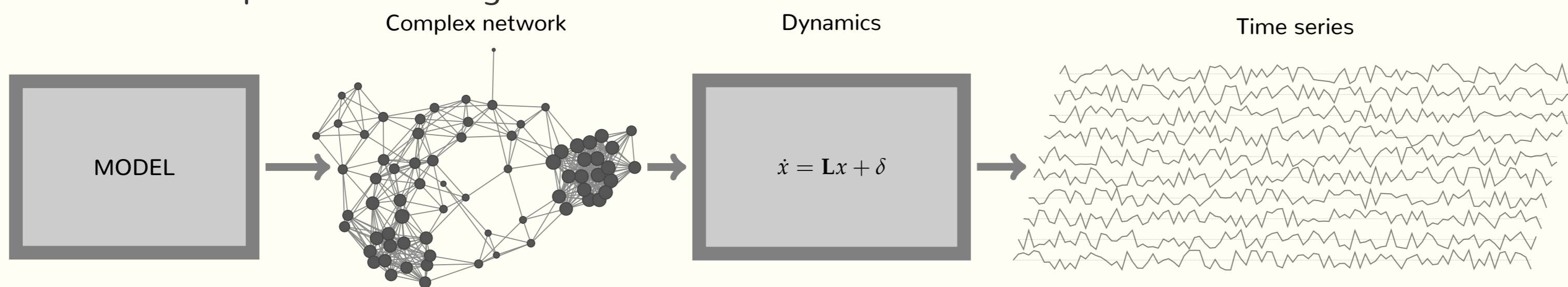
(performed on **BA** (see the plot), ER, RG graphs with similar results)
Synthetic data shows that the reconstructed network sensibly differs from the original.
Usually, F1 is maximized by a parameter value that returns a disconnected graph.
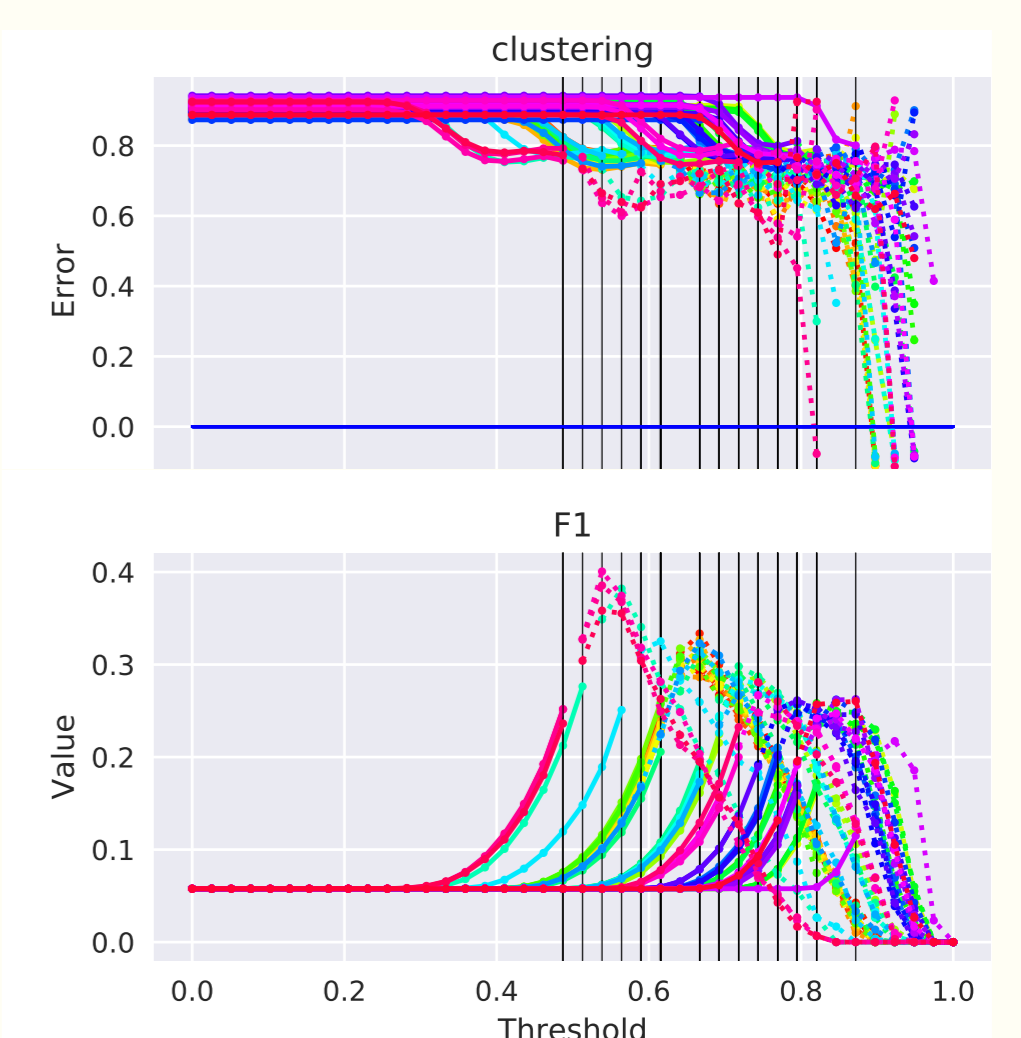


N: 200; Error: Observed - Real; Thresholding;
Each line one realization

## In the Literature

Reconstruction methods are often tuned to achieve an arbitrarily specified density [1], obtaining a single connected component and/or to achieve a certain property, such as being small-world *enough*. Analogously, in [2] the authors can set a hard threshold to recover the small-world property. In [3] the authors check three values for the regularization parameter of the graphical LASSO [4] sparsification approach, to reconstruct a dense, medium and sparse graph. In [5] the authors show that in certain cases the community structure of the reconstructed networks does not widely depend on the selected threshold value.

## References

[1] Lynall, Bassett, Kerwin, McKenna, Kitzbichler, Muller, Bullmore: Journal of Neuroscience **30**(28) (2010) 9477–9487

[2] Achard, Salvador, Whitcher, Suckling, Bullmore: Journal of Neuroscience **26**(1) (2006) 63–72

[3] Rosa, Portugal, Hahn, Fallgatter, Garrido, Shawe-Taylor, Mourao-Miranda: NeuroImage **105** (2015) 493 – 506

[4] Friedman, Hastie, Tibshirani: Biostatistics **9**(3) (2008) 432–441

[5] Yan, Jeub, Flammini, Radicchi, Fortunato: Phys. Rev. E **98** (Oct 2018) 042304